OSE Qualifying Examinations – 2006 Advanced Optics. Answer all questions. Begin each problem on a separate sheet. Write your Banner ID# and the Problem# on each sheet.

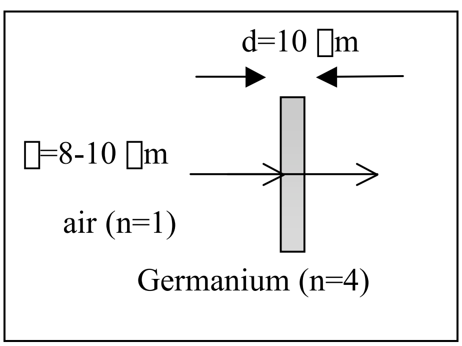
1. (5p) For wavelengths in the visible spectrum, the dispersion relation of a certain type of crown glass can be approximated by the relation

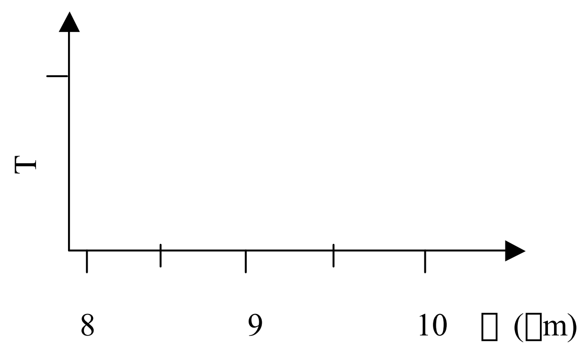
$$\omega(k) = 2 \times 10^8 k \,\square\, k^2$$

where ω is in sec^{-1} and k is in m^{-1}.
a) What is the phase velocity for light with λ=500 nm (in the glass)?
b) What is the wavelength of this same light if it emerges into vacuum?
c) What is the group velocity for a pulse centered at λ=500 nm (in the glass)?

2. (15p) We consider a spherical boundary, radius r, between air and a bulk glass with index of refraction $n = 1.5$.
a) Find the position of the focus in the glass, for a beam incident from the air, parallel to an axis through the center of curvature.
b) Find the position of the focus in the air, for a beam incident from the glass, parallel to an axis through the center of curvature.
c) What is the transfer matrix for paraxial rays entering the glass from air?

3. (20p) Light from an IR laser source (λ=8 to 10 μm) is incident on a Germanium parallel slab (d=10 μm, n=4) as shown.
Obtain the transmission versus wavelength, and plot it in the range of 8 to 10 μm at normal incidence. Be quantitative and give values for T_{max}, T_{min}, and finesse.

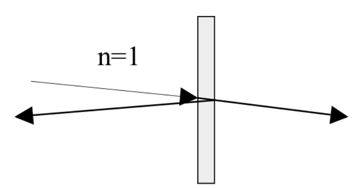




4. (20p) You are given an unknown semiconductor wafer with a thickness of 100 μm. A Nd:YAG laser beam (λ≈1 μm) with a power of 1 W is incident on this sample at near normal incidence (θ≈0). You measure a transmitted power of 10 μW and a reflected power of 200 mW.

Ignoring multiple reflections, obtain the complex refractive index ñ=n+iκ from the above measurements. Based on your results, explain why it was justified to ignore multiple reflections.

(Hints: make and justify simplifying approximations. It may be helpful to write down the general form for a plane wave moving through an absorbing medium.)



OSE Qualifying Examinations – 2006 General Questions. Answer all questions. Begin each problem on a separate sheet of paper. Put your Banner ID# and Problem# on each sheet.

1. (5p) A small ball bearing of mass m = 1 g is dropped from a height of 1 m. What is the wavelength of a photon with the same kinetic energy as the ball as it hits the ground? To which region of the electromagnetic spectrum does such a photon belong?

2. (5p) The atomic mass of Ar is 40. **Estimate** the Doppler broadening expected from an Ar ion laser if the effective gas temperature is 5000 K and the emission wavelength is 514 nm.

3. (10p) Consider a 30-cm long HeNe laser (633 nm) that operates on two adjacent longitudinal modes of equal amplitude *A* and identical polarization. The laser illuminates a photodiode with a 1-ns time resolution. The diode output is monitored on a 1-GHz oscilloscope. Derive an equation for the signal as a function of time. Sketch the oscilloscope trace that you expect and comment. Be as quantitative as possible. In particular, determine characteristic frequencies (if any).

4. (5p) Using the diffraction limited resolution of the human eye, find the longest distance at which one can distinguish the two headlights of an incoming car at night. Use reasonable assumptions for the car and human dimensions.

5. (5p) Explain why metals have high reflectivity, and why they appear a certain color.

6. (10p) Describe the physical principles of the operation of a commercially available laser of your choice, in 1 page or less.

7. (10p) Describe $\lambda/2$ and $\lambda/4$ waveplates. What is meant by the 'order' of a waveplate? What is the importance of the order when using short pulse lasers?

8. (5p) Describe the operation of a Fresnel rhomb and its application.

9. (5p) Laser light passes through two identical and parallel slits, 0.2mm apart. Interference fringes are seen on a screen, perpendicular to the slits, 1m away. The interference maxima are separated by 3.29 mm. What is the wavelength of the light used?

10. (10p) An object 3 cm high is placed 20 cm from (a) convex mirror and (b) a concave spherical mirror each of 20 cm radius of curvature. Determine the position and orientation of the image in each case.

11. (15p) An optical isolator is to be constructed to maximize the transmission in one direction (ideally T = 1) and to minimize the transmission in the opposite direction (ideally T = 0). To this end a slab of crown glass (d = 5 cm) is sandwiched between two linear polarizers set at 45^o with respect to each other and a longitudinal (parallel to the laser beam axis) magnetic field is applied. The Verdet constant of crown glass is $V = 1.6\ 10^{-12}$ arcmin/Tesla/m.

(a) In a few sentences explain how this isolator works.

(b) What magnetic field B_{opt} should be applied?

(c) What will the reverse transmission be if the magnetic field is 90% of this optimum value?

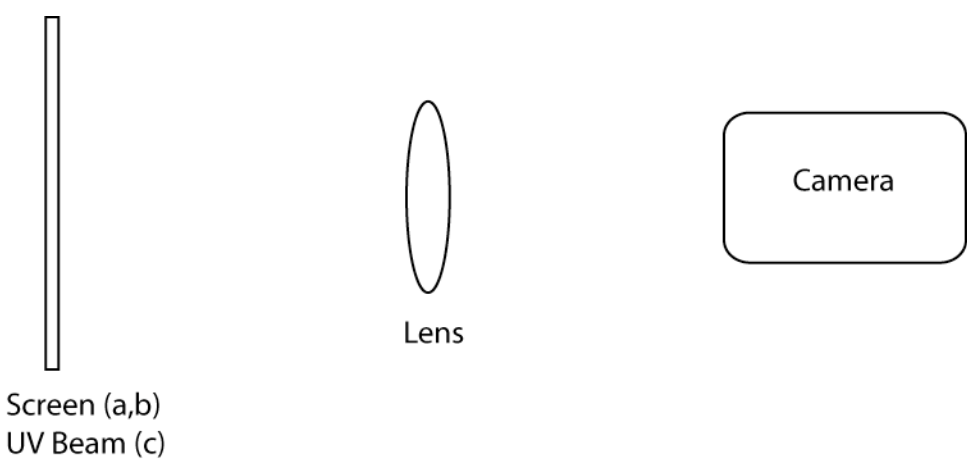
Hint: The Faraday effect states that the polarization of linearly polarized light is rotated by an angle $\theta = \pm VBd$. The sign of the rotation angle depends on the direction of the B field relative to the propagation direction of the beam.

12. (15p) A screen contains a small image, illuminated by 800 nm light, which you want to magnify 5 times. You are given a lens of 20 cm focal distance, 5 cm diameter, made of fused silica.

a) Find the relative positions of the lens and the image recorder (e.g. a CCD camera) that will give you the sharpest magnified image.

b) The pixels of the CCD are spaced by 10 μm. Is the resolution of the system limited by the CCD or by the lens?

c) You want to use the same optical system to observe the scattering of the cylindrical trace made in air by a narrow pencil of a UV beam (diameter 100 μm), substituted in the position of the screen. (The axis of the beam is in the plane where the screen used to be.) Will the image still be sharp on the CCD? If not, in which direction should you move the CCD to make a sharp image? Assume normal dispersion.



OSE Qualifying Examinations – 2006. E&M. Answer 3 of the 5 questions. Begin each problem on a separate sheet. Write your Banner ID# and the Problem# on each sheet.

1. A linearly polarized plane wave of (electric-field) amplitude E_0 traveling inside a semi-infinite dielectric of refractive index n is incident on its interface with air. The plane of polarization makes an angle of α with the plane of incidence, and the angle of incidence, θ, exceeds the critical angle for total internal reflection. Assume that the reflecting dielectric-air interface is planar.

a) Show that the totally reflected wave is in general elliptically polarized. In terms of the (complex) amplitude reflection coefficients, $r_\parallel$ and $r_\perp$ for the linear polarizations parallel and perpendicular to the plane of incidence, respectively, α, and the reflected polarization unit vectors $\hat{e}'_\parallel$ and $\hat{e}'_\perp$, write down the complex amplitude of the electric field vector of the reflected wave.

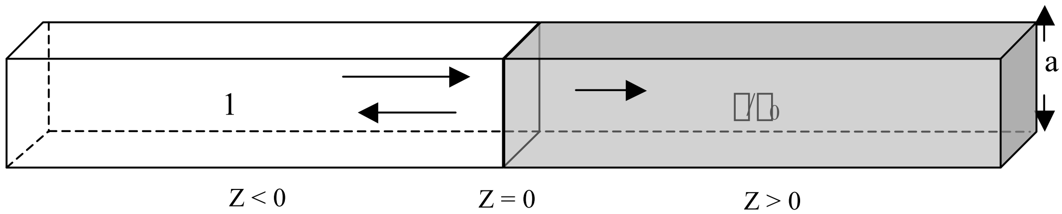
b) Noting that the argument of the complex quantity $(a+ib)/(a-ib)$, $a,b \in \mathbb{R}$, is 2arctan(b/a), show that the difference $\Delta\phi$ between the phases of the two polarization components of the reflected wave is

$$\Delta\phi = 2\arctan\left[\frac{\cos\theta\sqrt{n^2\sin^2\theta - 1}}{n\sin^2\theta}\right].$$

c) For what incidence angle θ_0, as a function of n, are the major and minor axes of the polarization ellipse of the reflected wave parallel and perpendicular to the plane of incidence?

d) Show that the condition of part c can only be achieved if $n > \sqrt{2}+1$.

2. Consider an infinitely long, hollow, conducting waveguide of square cross-section of side a. The axis of the waveguide defines the z axis. The right half of the waveguide, $z > 0$, is filled with a dielectric medium of constant ϵ/ϵ_0, while the left half, $z < 0$, contains vacuum. Consider a TM_{11} mode propagating in the $+z$ direction in the left half of the guide, and incident on the dielectric discontinuity at $z = 0$.



a) Write down the spatial distributions of the electric and magnetic fields of the incident TM mode in the left half of the guide.

b) Because of the dielectric discontinuity at $z = 0$, the incident mode is partially reflected and partially transmitted. By taking the amplitude reflection and transmission coefficients to be $\tilde{r}$ and $\tilde{t}$ respectively, write down the fields in the two halves of the guide. (*Hint:* Note that the constant γ^2 (*see formula sheet)* must be the same in both halves of the guide, because the lateral boundary conditions are identical in both halves. This means, since ϵ is different in the two halves, that the z-components of the wavevector must be different in the two halves. Thus, use the appropriate *k's* in the two halves. Also, you must replace k by $-k$ in all your waveguide formulas to determine the fields associated with the reflected wave in the left half.)

c) Determine $\tilde{r}$ and $\tilde{t}$ by applying the boundary conditions that the normal component of the D field and the tangential component of the E field are continuous across the dielectric interface.

3. In a source-free, free-space region the complex magnetic field is given by:

$$\vec{H} = j(\hat{a}_y - j\hat{a}_z)\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_0\, e^{-j\beta_o x + j\omega_0 t}$$

where E_o is a constant. Find:

a) The polarization of the wave

b) The sense of rotation, if any (clockwise or counter-clockwise), as a function of time. Justify your answer.

c) The associated electric field

d) The time-averaged power density

e) How would your answers to parts (a)-(d) change, ***quantitatively***, if the region of propagation were not free space and had a dielectric constant $\varepsilon/\varepsilon_0$?

4. Consider the motion of a charge q (mass m) in a static, cylindrically symmetric, quadrupole potential, $V(x, y, z) = U(x^2 + y^2 - 2z^2)$, where U is a positive constant.

a) Write down the differential equations for the motion of three spatial coordinates of the charge. Argue why the origin is a point of unstable equilibrium.

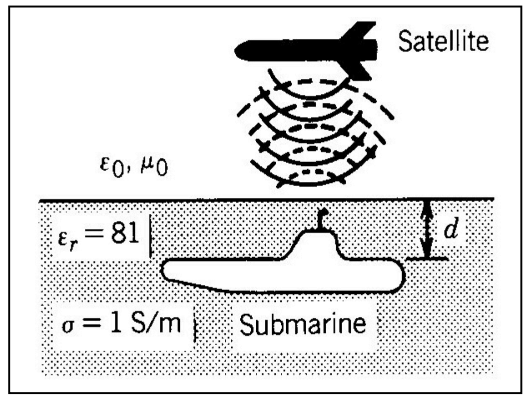
b) Ignoring the instability of equilibrium and assuming that the charge motion is confined to the *xy* plane for the duration of the experiment, show that circular motion is possible. What is the angular frequency, ω, of such motion in terms of U, q, and m?

c) Assume that the radius, a, of motion in part (b) is very small compared to $2\pi c/\omega$. The radiation from the charge can then be described accurately in the electric dipole approximation. Determine by means of simple physical arguments the nature of polarization of the far-field radiation emitted (i) in the plane of motion and (ii) normal to the plane of motion. Verify this by deriving an expression for the electric field of radiation emitted along an arbitrary direction of observation.

d) How does the expression for the electric field of radiation change when a perfectly conducting plane that is parallel to the plane of charge motion is brought close to the charge? Take the distance between the two planes to be *d*.

5. At large observation distances, the field radiated by a satellite antenna, which is attempting to communicate with a submerged submarine, is locally TEM (also assume a uniform plane wave), as shown in the figure below. Assuming the incident E-field before it impinges on the water (conductivity $\sigma = 1$ S/m $= 1$ mho/m and relative permittivity $\varepsilon_r = 81$) is 1 mV/m and the submarine is directly below the satellite, find at 1 MHz:

a) The intensity of the reflected radiation
b) The intensity of the transmitted radiation
c) The depth d (in meters) of the submarine at which it records the intensity of the transmitted radiation to be $1/e$ of the intensity just below the surface.
d) The time (in seconds) it takes the wave to travel from the surface of the ocean to the submarine at a depth of 100m.
e) The velocity of the electromagnetic wave in water.
f) How are the answers to parts (a) –(e) changed (qualitatively) if the frequency is increased from 1 MHz to 10 GHz?

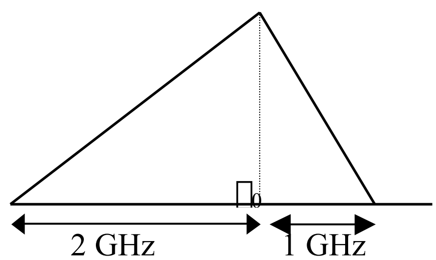


OSE Qualifying Examinations – 2006 Lasers. Answer all questions. Begin each problem on a separate sheet. Put your Banner ID# (last 5 digits) and the Problem# on each sheet.

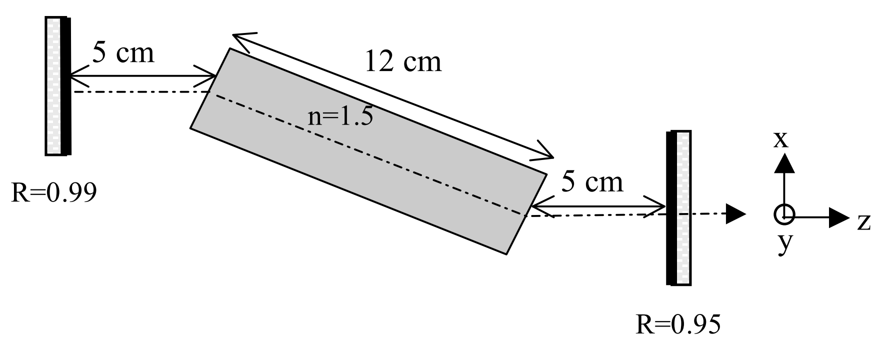
1. (5p) The rectangular faces of a semiconductor crystal often serve as laser mirrors. Assume that such a laser with a rectangular gain cross section produces radiation at 0.8 micrometers. If the minimum divergence of the beam is 10 arc min and the maximum divergence is 25 degrees, estimate the width of the exit face and the thickness of the active region.

2. (10p) The spontaneous lifetime of the upper laser level of Nd:glass is 300 microseconds. The population inversion density necessary for reaching the laser threshold is 10^{22} m^{-3}. **Estimate** the threshold pumping rate per unit volume (units m^{-3}) required to make the laser work (reach the threshold). You may assume that the lifetime of the lower laser level is very short (~ 0 s).

3. (5p) The *inhomogeneous* normalized lineshape function for a laser transition can be approximated by the graph below. What is the value of the lineshape function at $\nu=\nu_0$? Show your work.



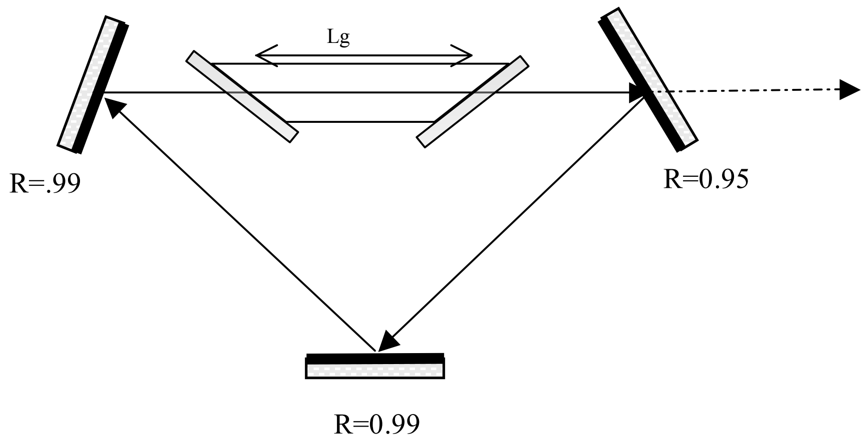
4. (15p) Estimate the photon lifetime of the passive cavity shown below, for the polarization that has the least loss. What is this polarization (x, y, z or a combination of these)? (The ends of the shaded rectangular block are tilted at 33.69° with respect to vertical.)



5. (15p) Consider a laser cavity of length *L* that consists of a flat outcoupler and an end mirror of radius of curvature *R*. The cavity is filled with a gas of refractive index *n*.

(a) Determine the values for R for which the resonator is stable.
(b) Determine the position and the size of the beam waist.
(c) Where should the end mirror be placed for the smallest beam waist? (Determine L.)
(d) The end mirror is replaced by a flat mirror. Find the radius of curvature of a glass lens (plano-convex, $n=n_1$) to be inserted in the cavity center so that the same beam waist as in part (b) is produced.

6. (15p) A unidirectional ring-cavity gas laser (with homogeneously broadened gain) is pumped 5 times above threshold to produce a cw output power of 10 Watts (the beam area inside the gain medium is 1 mm^2). What is the saturation intensity? What is the threshold integrated gain ($g_{th}=\gamma_{th}L_g$)?



7. (15p) A homogeneously broadened optical amplifier with a small-signal gain of 13 dB is irradiated with a wave with an intensity of 5 W/cm^2. The output intensity is 30 W/cm^2.

(a) Show that

$$\ln\frac{G}{G_0}+\frac{G-1}{I_s/I_{in}}=0$$

where $G_0=\exp(\gamma_0 L_g)$ is the small signal gain.

(b) What is the saturation intensity, I_s ?
(c) What is the maximum power (per unit area) extractable from this amplifier?

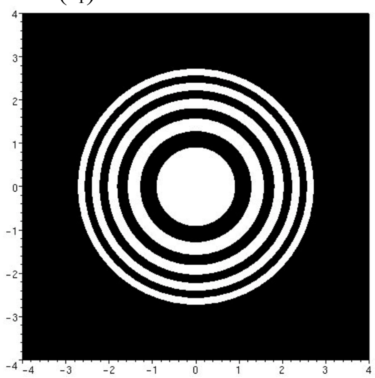
Note: $G(dB)=10\times\log_{10}(G)$, where $G=I_{out}/I_{in}$

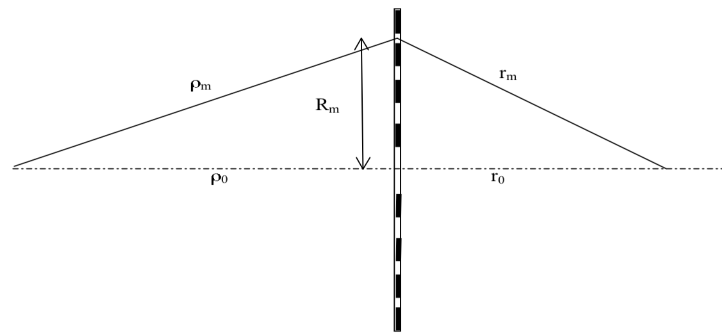
8. (20p) We consider a Fabry-Perot where the medium inside is a dye solution. The thickness of this Fabry-Perot is 1.0000 mm. The index of refraction of the liquid between the mirrors is $\tilde{n}=n+i\kappa$.

$n=1.33$; $\kappa=0$ for the un-pumped dye solution, while $\kappa>0$ for the pumped dye solution (indicating gain.) The reflectivity of the mirrors is $R=|r|^2=0.9$.

a) For the unpumped dye solution, find the exact resonance wavelength closest to 600 nm for which the Fabry-Perot has a maximum transmission. What is the value of that transmission factor?
b) Find the FWHM of the corresponding transmission peak, and the wavelengths for which the transmission is ½ of the peak value, using the high finesse approximation.
c) The dye solution is pumped by a green laser, to provide a gain such that $\kappa=4.55\times10^{-6}$. Find the transmission factor for the wavelengths at which the peak transmission of the *un-pumped* Fabry-Perot was ½ the peak value (i.e. for the wavelengths found in part b.)

5. (20p) Consider the Fresnel zone plate shown in Figure below, with all the odd Fresnel zones transparent and the even zones opaque. Derive the radius of the outer edge or the mth zone R_m ($<<r_0$, ρ_0) in terms of ρ_0 and r_0 and wavelength λ. Next write this quantity (R_m) in terms of the focal length of this lens (f_1).





6. (20p) Consider a Fabry-Perot interferometer (FPI) consisting of two parallel mirrors (M1 and M2) that form an air gap of d = 10 mm. One of the mirrors is mounted on a piezo-translation stage that is driven by a periodic voltage ramp. The transmission of the FPI is probed simultaneously by two lasers (laser b and c). A 3-channel oscilloscope displays the voltage applied to the piezo (channel a), the output of photodetector PD–b (channel b) and PD-c (channel c). Laser b can be considered a monochromatic light source with λ = 1 micron.

(a) What is the frequency of the saw-tooth voltage applied to the piezo?
(b) How much does the piezo move the mirror per one Volt applied (assuming a linear relationship between voltage and position)?
(c) Estimate the spectral resolution of your instrument.
(d) Estimate the spectral width of the radiation emitted by laser c. It is known that its spectrum is centered at 1000 nm.

Hint: The oscillographs are drawn to scale. Use a ruler.