Ph.D. Qualifying Examinations

General rules: All students seeking a Ph.D. are required to attempt the Written Qualifying Examinations after their first complete academic year; they are required to have passed all four examinations before beginning their third academic year. Each exam may be attempted at most twice* (see footnote below). Part-time students are not subject to the timetables above. The lowest passing score is 60%.

Exam details: The four exams consist of two common exams (common to all tracks) and two track-specific exams (for each track). All exams are offered in August; some track-specific exams (e.g., exams that are administered by the ECE department) may also be offered in January.

The exams that are common to all tracks are:

- Electromagnetics (1.5 hours, based on PHYC511 *or* the ECE555/ECE561 sequence)
- General Optics (3 hours; based on PHYC/ECE463 *and* PHYC476L)

The track-specific exams are:

Optical Sciences track

- Advanced Optics (1.5 hours; based on PHYC/ECE 554.)
- Lasers (1.5 hours; based on PHYC464 or ECE464)

Photonics track

- Semiconductor Optical Materials & Devices (based on ECE 570; this exam is administered by the ECE department)
- Semiconductor Physics (based on the ECE471/ECE572 sequence; this exam is administered by the ECE department)

Imaging Science track

- Stochastic Processes (based on ECE541; this exam is administered by the ECE department)
- Digital Image Processing (based on ECE533; this exam is administered by the ECE department)

Exit-examination requirement for MS Plan 2a: The Ph.D. qualifying examination will satisfy the exit-examination requirement for students enrolled in MS Plan 2a.

Grading out of the qualifying exam: The exam is waived for students who earn an average GPA of 4.0 or greater on *one* of the three sets of *four* OSE-required core courses depending upon the student's concentration of choice:

- **Optical Science track:** PHYC/ECE463, PHYC511 *or* ECE561, PHYC/ECE464, and PHYC/ECE 554
- **Photonics track:** PHYC/ECE463, PHYC511 *or* ECE561, ECE 570, and ECE 572
- **Imaging Science track:** PHYC/ECE463, PHYC511 *or* ECE561, ECE 533, and ECE 541

* PLEASE NOTE THAT THE ELECTROMAGNETICS, GENERAL OPTICS, ADVANCED OPTICS, AND LASER EXAMS ARE OFFERED ONLY ONCE PER YEAR. THUS, IF YOU WISH TO HAVE TWO ATTEMPTS AT THESE EXAMS, YOU MUST SIT FOR THE EXAMS BEFORE YOUR 2ND YEAR BEGINS.

OSE Qualifying Examination 2015 – Electromagnetics

Answer any 3 questions. Begin each question on a new sheet of paper. Put your banner ID at the top of each page. Staple all pages for each question together. Be sure to indicate what question # you are answering.

Possibly Useful Formulas

- Relation of spherical coordinates, (r, θ, ϕ), to Cartesian coordinates:

$$x = r\,\sin\theta\,\cos\phi, \quad y = r\,\sin\theta\,\sin\phi, \quad z = r\,\cos\theta.$$

  Unit vectors:

$$\hat{r} = \sin\theta\,\cos\phi\,\hat{x} + \sin\theta\,\sin\phi\,\hat{y} + \cos\theta\,\hat{z};$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}; \quad \hat{\theta} = \hat{\phi} \times \hat{r}.$$

- Laplacian in spherical coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}.$$

  and outside it is that due to an equivalent point magnetic dipole at the center.

- Time-averaged power radiated by an oscillating electric dipole:

$$P = \frac{\mu_0 |p|^2 \omega^4}{12\pi c}.$$

- Time-averaged power radiated by an oscillating magnetic dipole:

$$P = \frac{\mu_0 |m|^2 \omega^4}{12\pi c^3}.$$

- Magnetic field radiated by an oscillating magnetic dipole $\vec{m}\exp(-i\omega t)$ -

$$\vec{B} = \frac{\mu_0 k^2}{4\pi}(\hat{n}\times\vec{m})\times\hat{n}\frac{\exp(ikr - i\omega t)}{r}, \quad k = \omega/c.$$

- Instantaneous power radiated by a non-relativistically moving charge with acceleration a (Larmor formula):

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}.$$

- Fresnel formulas for the amplitude reflection coefficient of a plane wave incident at a planar interface between two dielectrics:

$$r_\perp = \frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'}; \quad r_\parallel = \frac{n'\cos\theta - n\cos\theta'}{n'\cos\theta + n\cos\theta'},$$

  where $\perp, \parallel$ refer, respectively, to polarizations perpendicular and parallel to the plane of incidence. The angles of incidence and refraction are θ and θ', and n, n' are the refractive indices of the medium of incidence and the medium of transmission, respectively.

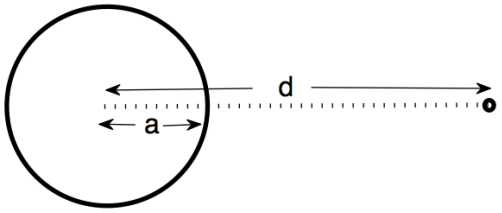
Question #1

1. Consider two different problems involving conducting and non-conducting bodies that are all held fixed, in which two different charge arrangements are placed on the bodies, denoted as $\rho_1(\mathbf{r})$, $\sigma_1(\mathbf{r})$ and $\rho_2(\mathbf{r})$, $\sigma_2(\mathbf{r})$, where ρ_1, ρ_2 and σ_1, σ_2 denote the volume and surface charge densities on the bodies in the two problems. The corresponding potentials are denoted as $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$, respectively.

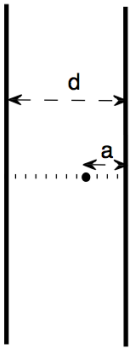
   (a) By expressing the potentials V_1, V_2 in terms of the charge densities, show that the following duality relation must hold:

$$\int d\tau \rho_1(\mathbf{r}) V_2(\mathbf{r}) + \oint dS\, \sigma_1(\mathbf{r}) V_2(\mathbf{r}) = \int d\tau \rho_2(\mathbf{r}) V_1(\mathbf{r}) + \oint dS\, \sigma_2(\mathbf{r}) V_1(\mathbf{r}),$$

   where $d\tau$ and dS are the volume and surface area elements, respectively.



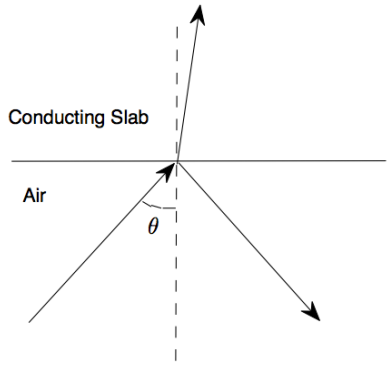
   (b) Use this relation for the arrangement of two conducting spheres, with radii a and b, that are a distance d ($d > a+b$) apart. Take b to be really small, so one of them is essentially a point-like body. By placing charges Q_1 on the large sphere and q_1 on the point sphere, while keeping the large sphere grounded, and then charge Q_2 on the large sphere and leaving the point sphere uncharged (*i.e.*, $q_2 = 0$), show that Q_1 must be equal to $-q_1 a/d$. Interpret this result physically.



(c) Apply a similar strategy to calculate the charge induced on a plate of a parallel-plate capacitor when a point charge q is placed a distance a from that plate and the plates, held a distance d apart, are kept grounded. Take the plates to be perfectly conducting. (*Hint:* Compare this charge distribution to that in which the two plates are charged to charge densities $\pm\sigma$ without any charge placed at the location of the point charge. Show that the potential in this case, $V_2(\mathbf{r})$, may be chosen to have a linear variation as a function of the distance from either plate and may be chosen to be zero at one of the plates.)

Question #2

2. A plane electromagnetic wave of angular frequency ω is incident at angle θ with respect to the normal of the surface of a semi-infinite slab occupying the region $z \geq 0$. Assume that the slab is highly conducting but non-magnetic, with a dieletric permittivity ϵ and conductivity σ that is large but not infinite, as shown in the figure.



(a) Show from Maxwell-Ampere's law that the slab may be regarded as a dielectric with a complex effective permittivity equal to $\epsilon + i\sigma/\omega \approx i\sigma/\omega$.

(b) Show that the wave transmitted into the conductor is an evanescent plane wave. What is the characteristic depth, in terms of σ, ω, and certain electromagnetic constants, to which the transmitted field propagates inside the slab in the high-conductivity limit, $\sigma >> \omega\epsilon$?

(c) Show that in the high conductivity limit, the transmitted wave propagates essentially normally to the surface, regardless of the angle of incidence.

(d) Using the Fresnel reflection formula, show that in the high-conductivity limit and for normal incidence, the fraction of the incident power reflected by the slab is $1 - 4\mathrm{Re}(n)/|n|^2$, where n is the effective index of refraction of the conducting slab? Express n in terms of σ, ω and electromagnetic constants.

(e) Use the result obtained in part (d) to calculate the fraction of incident power absorbed by the slab. Does your answer agree with the result that in the infinite-conductivity limit, no power is absorbed by the conductor?

Question #3

3. A small uncharged circular current loop of radius a carrying a constant current I_0 is initially horizontal. It is set into oscillatory motion about a diameter with an angular displacement about that diameter given by the time-dependent expression,

$$\theta(t) = \theta_0 \cos(\omega t),$$

as shown in the figure. Take $\omega a/c << 1$.

(a) Argue why the radiation from the oscillating current loop is predominantly of the magnetic-dipole variety.

(b) What is the state of wave polarization of radiation emitted along the vertical axis? Along the horizontal direction oriented at an angle ϕ to the fixed loop diameter? Take θ_0 to be small compared to 1, so approximations of the kind, $\sin\theta \approx \theta$, $\cos\theta \approx 1$, are valid.

(c) How much time-averaged power is radiated in the vertical direction per unit solid angle in the radiation zone?

(d) How does your answer to part (c) change when a perfect conductor with its flat horizontal surface is placed a distance d from the center of the oscillating loop? Take d to be arbitrary, when compared to the wavelength of radiation, but small compared to the distance to the observation point in the radiation zone? Show, in particular, that if d is varied then the power radiated in the vertical direction vanishes periodically as a function of d. Find an expression for the general value of d for which that happens. (*Hint*: Think of an image current loop which will radiate coherently with the given current loop.)

(e) Replace the perfect conductor by an infinitely permeable material. How does your answer to part (d) change?

Question #4

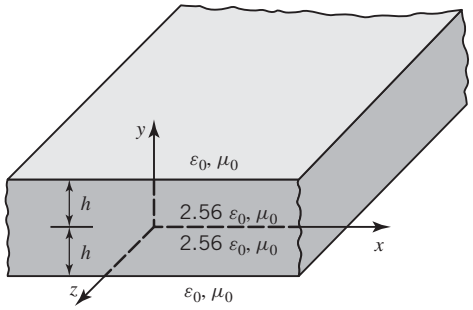
Consider a circularly polarized wave at $f = 10^8$ Hz propagating within a uniform, lossless dielectric medium. The wave encounters a planar interface with air at normal incidence. The functional form of the wave electric field at the interface is given as

$$\boldsymbol{E}^{\text{inc}}(z) = \left(\boldsymbol{a}_x - j\boldsymbol{a}_y\right)\exp(-j6\pi z)$$

where z in meters is in the direction normal to the interface. The interface is in the $x-y$ plane. Assume that the permeability in both regions is μ_0.

a) Draw a sketch describing the set-up of this problem and label as many things as you can (there should be at least 6 items labeled). [2 points]
b) Find the dielectric constant of the dielectric medium from which the circularly polarized wave emerges. [1 point]
c) Based on your answer to b) what is the dielectric medium? [1 point]
d) Find the reflection coefficients for both the $\boldsymbol{a}_x$ and $\boldsymbol{a}_y$ components. [1 point]
e) Find the transmission coefficients for both the $\boldsymbol{a}_x$ and $\boldsymbol{a}_y$ components. [1 point]
f) Find the polarization of the reflected field (is it still circular? or is it linear or elliptical? Justify your answer). [1 point]
g) Find the parity of part f) if it is rotating. [1 point]
h) Find the polarization of the transmitted field (is it still circular? or is it linear or elliptical? Justify your answer). [1 point]
i) Find the parity of part h) if it is rotating. [1 point]

Question #5



A dielectric slab of polystyrene ($\varepsilon = 2.56\varepsilon_0, \mu = \mu_0$) of height $2h$ is bounded above and below by free space, as shown in Figure 1. Assuming the time-harmonic instantaneous electric field within the slab is given by

$$\widetilde{\boldsymbol{E}}(x,t) = \left(\widehat{\boldsymbol{a}}_y 10 + \widehat{\boldsymbol{a}}_z 5\right)\cos(\omega t - \beta x)$$

where $\beta = \omega\sqrt{\mu_0\varepsilon}$, determine the:

(a) the complex spatial electric field intensity $\boldsymbol{E}(x)$ within the slab [1 point];
(b) the intrinsic impedance of the polystyrene [1 point];
(c) the time-harmonic instantaneous magnetic field within the slab [2 points];
(d) the corresponding complex spatial magnetic field intensity within the slab [1 point];
(e) the time-average Poynting vector (average power density) $\boldsymbol{S_{ave}}$ within the slab [2 points];
(f) the instantaneous electric and magnetic fields in free space right above and below the slab [2 points];
(g) the complex electric and magnetic fields in free space right above and below the slab [1 point].

Question #6

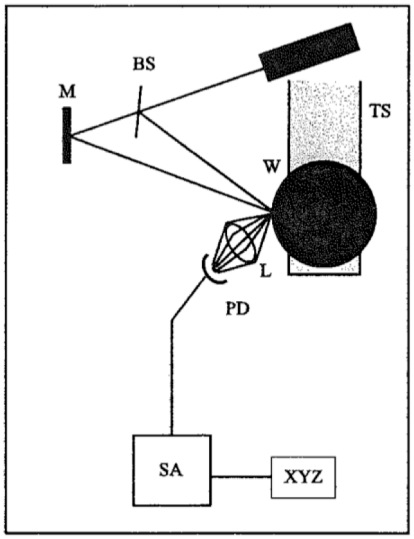
The electrical constitutive parameters of moist earth at a frequency of 1 MHz are $\sigma = 10^{-1}\ S/m$, $\varepsilon_r = 4$, and $\mu_r = 1$. Assuming that the electric field of a uniform plane wave at the interface (on the side of the earth) is $3\times10^{-2}\ V/m$, find the:

(a) Distance through which the wave must travel before the magnitude of the electric field reduces to $1.2\times10^{-2}\ V/m$ [1 point];
(b) Attenuation constant inside the earth (Nepers per meter) [1 point];
(c) Phase constant inside the earth (radians per meter) [1 point];
(d) Explain the physical meaning of propagation constant, attenuation constant and phase constant [2 points];
(e) Phase velocity inside the earth (in meters per second) [1 point];
(f) Wavelength inside the earth (in meters) [1 point];
(g) Intrinsic impedance of the earth [1 point];
(h) The expressions for the instantaneous electric and magnetic fields inside the earth, assuming the plane wave propagates in the $+\hat{\boldsymbol{z}}$ direction and the electric field is in the $+\hat{\boldsymbol{x}}$ direction [2 points].

General Optics PhD Qualifying Examination 2015

Answer all questions. Begin each question on a new sheet of paper.
Put your Banner ID at the top of each page, and staple the pages (if more than one) for each question together.

1a. It is possible to measure the rotation speed of a wheel that has reflective tape on its perimeter with the set up at right, taken from the Optics lab manual.



The rectangle at the top is a HeNe laser, wavelength 632.8 nm. It is split into two equally intense beams using beamsplitter BS. The beams are made to intersect on the perimeter of the wheel W. Lens L and photodiode PD detect scattered light from the wheel.

If the lower beam has an incident angle of 20° above the normal, and the upper beam has an incident angle of 30° above the normal, what is the spacing of the interference pattern formed on the wheel?

1b. For a setup where the fringe spacing is 6 microns (maximum to maximum), a peak in the spectrum analyzer is detected at 3 kHz. The wheel diameter is 14 cm. What is the speed of the perimeter, and the angular speed of the wheel?

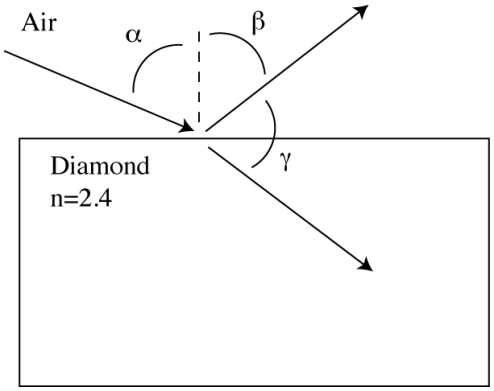
1c. A student wishes to make the lower beam impinge on the wheel at normal incidence, so he moves the mirror M very far to the left, a distance of a few meters. He notices that the signal peak moves to a different frequency, but also gets very weak. Does it move up or down in frequency? Why does the signal become very weak?

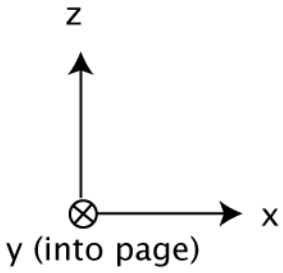
2. An astronaut is looking at a point source in a training pool (filled with water, n=1.33). The source is 200 cm from the vertex of the transparent hemispherical shield in front of the helmet, Where is the image of the point source that the astronaut sees? (The shield has a radius of 20 cm and is thin and of uniform thickness.)

3. Unpolarized light in air is incident on a flat surface of a diamond. The reflected light is completely polarized.

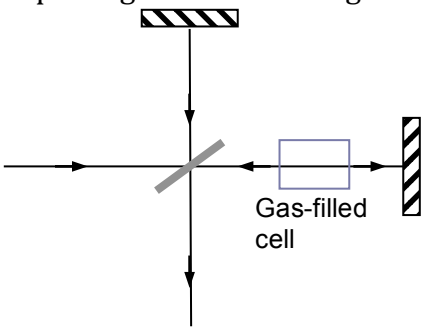
a. To the nearest degree, what are the angles α, β, γ in the figure?

b. What is the nature and orientation of the polarization (electric field orientation) for the reflected light? Express in terms of the coordinate system given below.

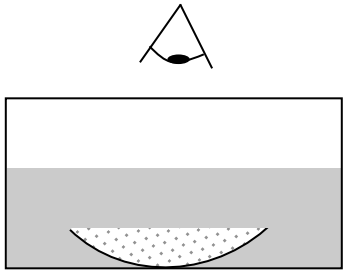




4. A gas-filled cell of length 5 cm is inserted in one arm of a Michelson interferometer, as shown in the figure below. The interferometer is in vacuum and is illuminated by light of wavelength 500 nm. As the gas is evacuated from the cell, 40 fringes cross a point in the field of view, Estimate the refractive index of this gas (the thickness of the splitting mirror can be ignored).



5. A plano-convex lens with a refractive index of 1.5 and power of 0.1 diopter (in air) is placed, convex surface down at bottom of a glass container (that is optically flat) filled with water (n = 1.33). Using a microscope and a sodium lamp (λ = 590 nm), we observe interference fringes from top. Determine the radius of the first dark ring.



6. Ice is birefringent, with indices 1.309 and 1.313.
a. If you want to make a "zero-order" quarter wave plate for a HeNe laser (wavelength 632.8 nm in vacuum), what would be its thickness?

b. Suppose you make a quarter wave plate with a nominal thickness of 1 mm. Estimate its "bandwidth." In other words, what is the vacuum wavelength nearest to 632.8 for which the ice waveplate will give linearly polarized output (rather than the desired circularly polarized output) for input linearly polarized at 45° to the optical axis? An approximate answer is sufficient.

7. You have 2 microscope objectives: Objective A is 100X magnification, with a numerical aperature of 0.4. Objective B is 40X magnification, with a numerical aperture of 0.8.

a. Estimate the resolution of each. Which has better resolving power?
b. These objectives were designed to be used with a microscope with tube length 25 cm (i.e. they place an image 25 cm from the objective.) Approximating these as simple lenses, what are their focal length and diameters?

8. A solar sail is to be used to suspend a spacecraft at a distance from the Sun equal to Earth's, by reflecting solar photons. It is 1.5×10^{11} m from Earth to the Sun. The Sun radiates 3.9×10^{26} W. It is proposed to make the sail out of aluminum foil, which has a density of 2.7 g/cm^3.

How thin does the foil have to be? (The spacecraft is quite far from the Earth, so you may ignore gravitational force from the Earth and other planets.)

9. A laser at 488 nm provides a 100 W TEM_{00} Gaussian beam with a $1/e^2$ waist radius of w_0 = 1 mm at the output coupler.
a. Estimate the approximate size of the spot this laser would shine on the moon (distance = 400,000 km).
b. If there is a corner cube on the moon to retro-reflect the light, estimate the number of photons per second detected by a 1 cm-square detector on Earth.

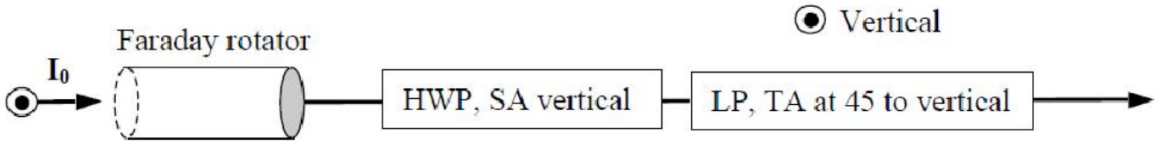
10.
a. What is the physical meaning of phase velocity and group velocity?
b. Describe two methods for mode-locking a laser
c. Describe two methods for Q-switching a laser
d. What waveplate would you use to change horizontal polarization to vertical, without loss?
e. Explain why Brewster windows are sometimes used in lasers?
f. What should be the orientation of the transmission axis on polarized sunglasses?

Advanced Optics PhD Qualifying Examination – August, 2015
Answer all questions. All count equally. Begin each question on a new sheet of paper, and staple all pages for each question together. Put your banner ID# at the top of each page.

Q1.
Monochromatic linearly polarized light passes through a Faraday rotator, a half-wave plate (HWP), and finally a linear polarizer (LP). The original polarization and the slow axis (SA) of the HWP are vertical (perpendicular to the page) while the transmission axis of the last polarizer is 45 degrees to vertical. The Faraday cell is 10 cm long with a Verdet constant of 0.0161 min/G-cm and a uniform magnetic field B is applied on the Faraday cell.

a) Show the orientation of the magnetic field for maximum rotation.
What is the polarization state of the rotator output if B=10kG?

b) At what magnetic field strength the intensity of the output beam is $I_0/4$ (where I_0 is the irradiance of the input beam)?



Q2. Design two Fresnel lenses of a little less than 2 mm radius, for a focal distance of 80 cm, at a wavelength of 0.8 μm.

a) What are the radii of the zones?

b) The first lens is made by blocking selected zones. Which ones should be blocked?

c) The second lens is to be made by applying a phase shift to selected zones. What phase shift and what zones?

d) Calculate the field and intensity at the focus for the two lenses, each illuminated by a plane wave of amplitude E_0.

e) Compare your answer to (d) to the intensity at the focus of a perfect lens of the same focal length (80 cm) illuminated by a Gaussian beam of w = 2 mm.

Hint: the optical field generated by the entire unobstructed wavefront is equal to one-half the contribution from the first zone.

Q3. You are asked to select a square flat grating 10 cm wide, for best wavelength selection (in first order) at 600 nm. The clearance is such that the acceptance angle for the incident beam ranges from -89° to +89°.

a) What is the configuration (angle of incidence and diffraction) and groove density that would give you the best resolution?

b) Assuming you can resolve a deviation angle of 1°, what is the corresponding resolution in wavelength $\Delta\lambda_1$?

c) What is the diffracted angle (first order) at the second harmonic (300 nm) and resolution $\Delta\lambda_2$ corresponding to a deviation angle of 1°?

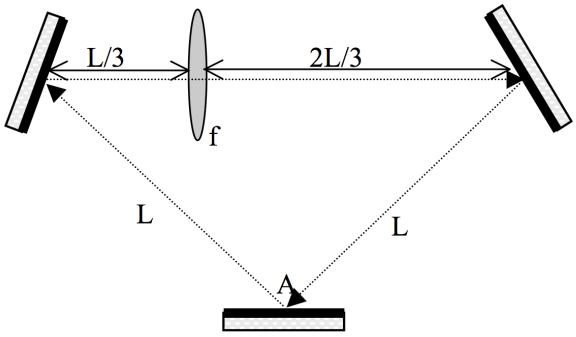
d) Can you achieve the same resolution at 300 nm (rather than 600 nm) in higher order? If so, which order?

Laser Physics PhD Qualifying Examination 2015

Answer all questions. Begin each question on a new sheet. Put your Banner ID on each page. Staple all pages for each question (separately) together.

1.

Consider the ring cavity with flat mirrors shown below:



(a) Starting from point A and assuming clockwise propagation, write the matrix product needed to obtain the ABCD matrix of the cavity.

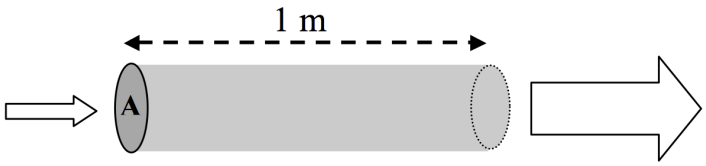
(b) Where is the beam waist in the cavity? Repeat (a), but start from the position of the beam waist.

(c) Find the ABCD system matrix of the cavity using (a).

(d) Find the values of *f* for which the cavity is stable.

2.

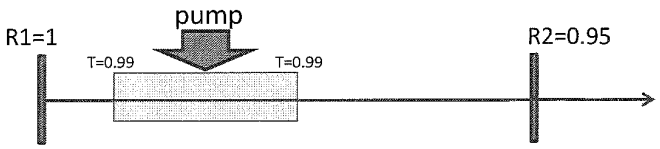
A homogeneously broadened optical amplifier with cross-sectional area of A= 0.5 cm^2 and saturation intensity of 20 W/cm^2 has small signal gain of 10 dB.



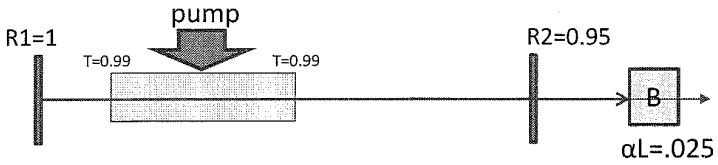
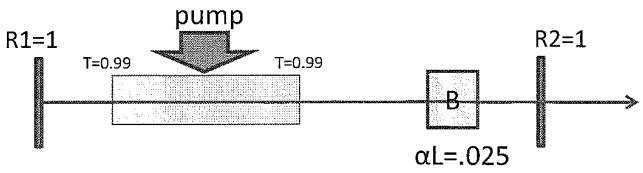
(a) Calculate the small signal gain coefficient (γ_0).
(b) Estimate the maximum power that can be extracted from this amplifier
(c) If the input power is such that the gain is suppressed by a factor of 2 (from its small signal value), compute the output power.

3.

(a) Calculate the integrated threshold gain ($g_{th}=\gamma_{th}L_g$) for the system below.



(b) For $g_0=5g_{th}$, calculate the *output power* (under proper approximation), and the *power absorbed in the cell B* having an absorbance ($\alpha \times L$) of 0.025. Assume homogenously broadened gain with saturation power $P_s=3W$.



(c) The absorber cell B is now placed inside the cavity, and mirror 2 is replaced with one having R2=1. What is the new threshold? If pumped 5x above the threshold, what is the absorbed power in cell B?