Ray tracing matrices for $\begin{bmatrix} y \\ \alpha \end{bmatrix}$

Free space: $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ Lens $\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$ Curved Interface $n_1 - -n_2$: $\begin{pmatrix} 1 & 0 \\ \frac{n_1-n_2}{Rn_2} & \frac{n_1}{n_2} \end{pmatrix}$

Snell's law: $n_i \sin\theta_i = n_t \sin\theta_t$ Refraction at spherical interface: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2-n_1}{R}$

Grating equation: $\sin\theta' - \sin\theta = m\frac{\lambda}{d}$ Lens-makers's formula: $\frac{1}{f} = (\frac{n_2-n_1}{n_1})\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Gaussian beams

$$\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi n w^2} = \frac{1}{R} - \frac{i}{\rho}$$

$\rho = \rho_0\left(1 + \frac{z^2}{\rho_0^2}\right)$ $R = z\left(1 + \frac{\rho_0^2}{z^2}\right)$ $\rho_0 = \frac{\pi n w_0^2}{\lambda}$ $q_2 = \frac{Aq_1+B}{Cq_1+D}$

Phase transformation of a spherical lens: $\phi(x,y) = \frac{k}{2f}(x^2+y^2)$

$$\tilde{\mathcal{E}}_{mp}(x,y,z) = \mathcal{E}_0 H_m(\frac{\sqrt{2}x}{w})H_p(\frac{\sqrt{2}x}{w})\frac{w_0}{w}e^{-i\frac{kr^2}{2q}}e^{-i[kz-(1+m+p)\arctan z/\rho_0]}$$

For a cavity: $\frac{1}{q} = \frac{D-A}{2B} \mp \frac{i}{2B}\sqrt{4-(A+D)^2}$

Fresnel equations

$$r_\parallel = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i} \qquad r_\perp = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$$

$$t_\parallel = \frac{2\sin\theta_i\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} \qquad t_\perp = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$

Fabry-Perot *Field* transmission:

$$\mathcal{T} = \frac{(1-R)e^{-i\vec{k}\cdot\vec{d}}}{1-Re^{i\delta}} \qquad \delta = 2\varphi_r - 2\vec{k}\cdot\vec{d} = 2\varphi_r - 2kd\cos\theta |\mathcal{T}|^2 = \frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

Finesse $F = \frac{\pi\sqrt{R}}{1-R}$ Photon lifetime: $\tau = \frac{\tau_{rt}}{\delta_c} = \frac{Q}{\omega}$ $\delta_c = \Sigma$ gain/losses $= 2\alpha L + \ln\left(\frac{1}{R_1R_2}\right)$

Fringe visibility $V = \frac{2\sqrt{I_1I_2}|\gamma(\tau)|}{(I_1+I_2)}$ **Irradiance** $I = < S > = \frac{1}{2\sqrt{\mu_0/\epsilon}}\mathcal{E}_0^2 = \frac{nc\epsilon_0}{2}\mathcal{E}_0^2$

Faraday cell polarization rotation $\beta = VBd$ **Kerr electrooptic birefringence** $\Delta n = K\mathcal{E}^2\lambda$

Linear electrooptic effect $\Delta n = n^3\frac{r}{2}E$

Diffraction **diffraction integral** z = distance between aperture plane (ξ,η) and observation plane (x,y)

$$U_D(x,y) = \frac{ie^{-ikz}}{\lambda z}\int\int_{-\infty}^{\infty} U_A(\xi,\eta)e^{\left\{\frac{-ik}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]\right\}}d\xi x\eta$$

Fourier transforms $g(\Omega) = \int_{-\infty}^{\infty} f(t)e^{i\Omega t}d\Omega$ $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(\Omega)e^{-i\Omega t}d\Omega$

Blackbody radiation Energy density: $\rho(\nu)d\nu = \frac{8\pi h n^3\nu^3 d\nu}{c^3}\frac{1}{e^{h\nu/kT}-1}$ Power $P = \sigma AT^4$

Thermal equilibrium $N_2/N_1 = g_2/g_1\exp[-(E_2-E_1)/kT]$

Einstein coefficients $A_{21} = 1/T_1$ $B_{21} = \frac{c^3}{8\pi n^3 h\nu^3}A_{21}$ $g_2B_{21} = g_1B_{12}$

Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$ with $\int g(\nu)d\nu = 1$ Gain/loss (2-level): $\alpha_0 = \sigma\left[N_2 - \frac{g_2}{g_1}N_1\right]$

Gain (absorption): $\frac{dI}{dz} = (-)\frac{\alpha_0 I}{1+I/I_s}$ Integration: $\ln\frac{I(z,t)}{I_0(t)} + \frac{I(z,t)-I_0(t)}{I_s} = \alpha_0\ell$

Saturation energy density $W_s = h\nu/(2\sigma)$ Saturation intensity: $I_s = W_s/T_1$

Maxwell's equations

$$
\begin{aligned}
\nabla \cdot \vec{D} &= \rho \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{H} &= J + \frac{\partial \vec{D}}{\partial t} \\
\nabla \cdot \vec{B} &= 0
\end{aligned}
$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

Curl identity: $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

Vector Potentials: $\vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$ $\quad \vec{E} = -\frac{1}{\epsilon} \nabla \times \vec{F}$ $\quad$ Complex Poynting vector $\quad \vec{S} = \frac{1}{2}(\vec{E} \times \vec{H}^*)$

Biot Savart $dB = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \vec{x}}{|\vec{x}|^3}$

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ $\quad \vec{E} = -\nabla \phi$ $\quad$ Force on magnetic dipole $\vec{m}$: $\vec{F} = (\vec{m} \cdot \nabla)\vec{B}$ $\quad$ Torque: $\vec{\tau} = \vec{m} \times \vec{B}$

Magnetic induction $\vec{B}$ due to a magnetic dipole $\vec{m}$:

Near field:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{3(\vec{m} \cdot \vec{n})\vec{n} - \vec{m}}{r^3}, \quad \vec{n} = \frac{\vec{r}}{r}$$

Far field;

$$\vec{B} = \frac{k^2 \mu_0}{4\pi} \frac{e^{ikr}}{r} (\hat{n} \times \vec{m}) \times \hat{n}$$

Dipole Radiation

$\vec{H} = \frac{ck^2}{4\pi} (\hat{n} \times \vec{p}) \frac{e^{ikr}}{r}$

$\vec{E} = \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{H} \times \hat{n}$

Waveguides

$$\vec{H}_t = \frac{\pm 1}{Z} \hat{z} \times \vec{E}_t \qquad Z = \left\{ \begin{array}{lll} \frac{k}{\epsilon \omega} & \text{(TM)} & \vec{E}_t = \pm \frac{ik}{\gamma^2} \nabla_t \psi \\ \frac{\mu \omega}{k} & \text{(TE)} & \vec{H}_t = \pm \frac{ik}{\gamma^2} \nabla_t \psi \end{array} \right\} \qquad (\nabla_t^2 + \gamma^2)\psi = 0 \qquad \gamma^2 = \mu \epsilon \omega^2 - k^2$$

where $\psi e^{\pm ikz}$ is $E_z(H_z)$ for TM(TE) waves