



EQUATION SHEET (OPTICS & LASER)

>> Ray tracing matrices:

$$\text{Thin lens: } \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \quad \text{Free space: } \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \quad \text{Curved interface: } \begin{bmatrix} 1 & 0 \\ \frac{n_1-n_2}{Rn_2} & \frac{n_1}{n_2} \end{bmatrix} \quad (R: \text{radius})$$

$$>> \text{Snell's law: } n_i \sin \theta_i = n_t \sin \theta_t \quad \text{Lens-makers's formula: } \frac{1}{f} = \left(\frac{n_2-n_1}{n_1} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (R_i: \text{radius})$$

$$>> \text{Grating equation: } \sin \theta' - \sin \theta = m \frac{\lambda}{d}$$

$$>> \text{Gaussian beam: } \frac{1}{q} = \frac{1}{R(z)} - \frac{j\lambda_0}{n\pi w^2(z)} \quad w(z)^2 = w_0^2 \left(1 + \frac{z^2}{z_0^2} \right), \quad R(z) = z \left(1 + \frac{z_0^2}{z^2} \right), \quad z_0 = \frac{\pi n w_0^2}{\lambda_0}$$

$$\frac{E(x, y, z)}{E_{m,p}} = H_m \left[\frac{\sqrt{2} x}{w(z)} \right] H_p \left[\frac{\sqrt{2} y}{w(z)} \right] \times \frac{w_0}{w(z)} \exp \left[-\frac{x^2 + y^2}{w^2(z)} \right] \times \exp \left\{ -j \left[kz - (1+m+p) \tan^{-1} \left(\frac{z}{z_0} \right) \right] \right\} \times \exp \left[-j \frac{kr^2}{2R(z)} \right]$$

$$>> \text{Cavity stability condition: } 0 \leq \frac{A+D+2}{4} \leq 1$$

$$>> \text{Gaussian beam propagation from point 1 to point 2: } q_2 = \frac{Aq_1+B}{Cq_1+D}$$

$$>> \text{Gaussian beam in a cavity: } \frac{1}{q} = -\frac{A-D}{2B} - j \frac{\sqrt{1-\left(\frac{A+D}{2}\right)^2}}{B}$$

$$>> \text{Fabry-Perot Transmisison and Reflection (when } r_1=r_2=r): t = \frac{E_t}{E_i} = \frac{(1-r^2)e^{-i\delta/2}}{1-r^2e^{-i\delta}}, \quad r = \frac{E_r}{E_i} = \frac{r^2(e^{-i\delta}-1)}{1-r^2e^{-i\delta}}, \quad \delta = 2kd$$

$$T = \left| \frac{E_t}{E_i} \right|^2 = \frac{(1-r^2)^2}{1+r^4-2r^2\cos(\delta)} \quad \text{and} \quad R = \left| \frac{E_r}{E_i} \right|^2 = 1-T$$

r is reflection coefficient and $R (= r^2)$ is reflectance. For asymmetric case ($r_1 \neq r_2$): $r^2 \rightarrow r_1 \times r_2$

$$\text{Finess: } F = \frac{\Delta\nu_{FSR}}{2\Delta\nu_{1/2}} = \frac{\pi r}{1-r^2}, \quad \text{Photon life time: } \tau = \frac{\tau_r}{\delta_c} = \frac{Q}{\omega_0} = \frac{\tau_{RT}}{1-S} = \frac{2nd/c}{1-R^2}, \quad (\text{note that } 2\Delta\nu_{1/2} \text{ is the FWHM})$$

$$\text{Free spectral range: } \Delta\nu_{FSR} = \frac{c}{2nd}, \quad \text{Quality fator: } Q = \frac{\nu}{\nu_{FSR}} \times F = \frac{\nu}{2\Delta\nu_{1/2}}$$

$$>> \text{Fringe visibility (two path inteference with single source): } V = \frac{2|\gamma(\tau)|\sqrt{I_1 I_2}}{I_1 + I_2}, \quad \gamma(\tau): \text{complex degree of temporal coherence}$$

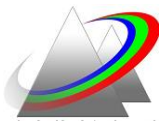
$$>> \text{Faraday cell rotation: } \beta = VBd, \quad \text{Kerr effect: } \Delta n = KE^2\lambda, \quad \text{Pokels effect: } \Delta n = n^3 \left(\frac{r_{eo}}{2} \right) E$$

$$>> \text{Irradiance: } I = \langle S \rangle = \frac{1}{2 \times \sqrt{\mu_0/\epsilon}} E_0^2 = \frac{nc\epsilon_0}{2} E_0^2$$

>> Fresnel equations:

$$r_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_t + n_t \cos \theta_i} \quad \left\| \quad r_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \right.$$

$$t_{\parallel} = \frac{2 \sin \theta_i \cos \theta_t}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i} \quad \left\| \quad t_{\perp} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t)} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t} \right.$$



EQUATION SHEET

OPTICS (continued)

>> **Fresnel integral:** z =distance between aperture plane (ξ, η) and observation plane (x, y) :

$$U_p(x, y) = \frac{ie^{-ikz}}{\lambda z} e^{(-ik/2z)(x^2+y^2)} \iint_{-\infty}^{+\infty} U_A(\xi, \eta) \times e^{(-ik/2z)(\xi^2+\eta^2)} \times e^{-i(2\pi/\lambda z)(x\xi+y\eta)} d\xi d\eta$$

>> **Phase transformation of a spherical lens:** $t_l(x, y) = \exp\left[\frac{ik}{2f}(x^2 + y^2)\right]$

>> **Fourier Transforms:** $\mathcal{F}\{g(x)\} = \int_{-\infty}^{\infty} g(x)e^{i2\pi(xf_x)} dx$, $\mathcal{F}^{-1}\{G(f_x)\} = \int_{-\infty}^{\infty} G(f_x)e^{-i2\pi(xf_x)} df_x$, $f_x = \frac{x}{\lambda z}$
(between space and spatial frequency domain)

>> **Free space propagation (frequency domain):** $\mathcal{F}\{U(x)\} = \mathcal{F}\{U(\xi)\} \times \exp(i\frac{2\pi^2}{k}zf_x)$

Jones matrices:

$$\text{Linear polarizer : TA horizontal: } \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} , \quad \text{TA vertical } \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Phase retarders : } \begin{bmatrix} e^{i\delta_x} & 0 \\ 0 & e^{i\delta_y} \end{bmatrix} , \quad \text{Rotator : } \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$$

Doppler shift when $v \ll c$: $\frac{\lambda'}{\lambda} \approx 1 - \frac{v}{c}$

Radiation pressure (when a wave is completely absorbed): $P = \frac{\langle S \rangle}{c} = \frac{I_{inc}}{c}$

LASER

Einstein coefficients: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$, $g_2 B_{21} = g_1 B_{12}$, $\frac{1}{\tau_{21,rad}} = A_{21}$

Natural linewidth: $g(\nu) = \frac{\Delta\nu}{2\pi[(\nu_0 - \nu)^2 + (\frac{\Delta\nu}{2})^2]}$, $\Delta\nu = \frac{1}{2\pi}(\frac{1}{\tau_2} + \frac{1}{\tau_1}) = \frac{1}{2\pi}(A_1 + A_2)$, $A_2 = \sum_{j < 2} A_{2j}$

Stimulated emission cross section: $\sigma(\nu) = A_{21} \frac{\lambda^2}{8\pi n^2} g(\nu)$

Gain (loss) coefficient for a two-level system: $\gamma(\nu) = \sigma(\nu) \left[N_2 - \frac{g_2}{g_1} N_1 \right]$

Gain/Absorption: $\frac{dI_\nu}{dz} = \frac{\gamma_0 I_\nu}{1 + \bar{g}(\nu)(I_\nu/I_s)}$, $\ln\left(\frac{I_{out}}{G_0 I_{in}}\right) = \frac{I_{in} - I_{out}}{I_s}$ where $G_0 = e^{\gamma_0 l_g}$

Saturation intensity: $I_s = \frac{h\nu}{\sigma(\nu)\tau_2}$

CONSTANTS

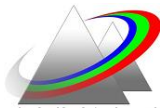
$$h=6.62\times10^{-34}\text{ J.s}$$

$$c=3\times10^8\text{ m/s}$$

$$e=1.6\times10^{-19}\text{ C}$$

$$k=1.38\times10^{-23}\text{ J.K}^{-1}$$

$$\varepsilon_0=8.85\times10^{-12}\text{ F/m}$$



EQUATION SHEET

ELECTROMAGNETISM

Possibly Useful Formulas

- Relation of spherical coordinates, (r, θ, ϕ) , to Cartesian coordinates:

$$x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta.$$

Unit vectors:

$$\hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z};$$

$$\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}; \quad \hat{\theta} = \hat{\phi} \times \hat{r}.$$

- Electric potential at position $\vec{r}$ due to a point electric dipole of moment $p\hat{z}$ located at the origin:

$$V(\vec{r}) = \frac{\vec{p} \cdot \vec{r}}{4\pi\epsilon_0 r^3}.$$

- Polarization induced in a dielectric sphere of dielectric permittivity, ϵ , by a uniform external field, $\vec{E}$:

$$\vec{P} = 3\epsilon_0 \left(\frac{\epsilon_r - 1}{\epsilon_r + 2} \right) \vec{E}, \quad \epsilon_r \equiv \frac{\epsilon}{\epsilon_0}.$$

- Time-averaged power radiated by an oscillating electric dipole:

$$P = \frac{\mu_0 |\vec{p}|^2 \omega^4}{12\pi c}.$$

- Time-averaged power radiated by an oscillating magnetic dipole:

$$P = \frac{\mu_0 |\vec{m}|^2 \omega^4}{12\pi c^3}.$$

- Electric and magnetic fields at point $\vec{r}$ due to an oscillating electric dipole of moment $\vec{p} \exp(-i\omega t)$ at the origin -

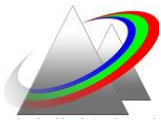
$$\vec{E} = \frac{e^{i(kr - \omega t)}}{4\pi\epsilon_0} \left\{ k^2 \frac{(\hat{r} \times \vec{p}) \times \hat{r}}{r} + \left(\frac{1}{r^3} - \frac{ik}{r^2} \right) [3(\hat{r} \cdot \vec{p})\hat{r} - \vec{p}] \right\};$$

$$\vec{B} = \frac{k^2 e^{i(kr - \omega t)}}{4\pi c \epsilon_0} (\hat{r} \times \vec{p}) \left(\frac{1}{r} + \frac{i}{kr^2} \right); \quad k = \frac{\omega}{c}; \quad \hat{r} = \frac{\vec{r}}{r}$$

- Fresnel formulas for the amplitude reflection coefficient of a plane wave incident at a planar interface between two dielectrics:

$$r_{\perp} = \frac{n \cos \theta - n' \cos \theta'}{n \cos \theta + n' \cos \theta'}; \quad r_{\parallel} = \frac{n' \cos \theta - n \cos \theta'}{n' \cos \theta + n \cos \theta'},$$

where $\perp, \parallel$ refer, respectively, to polarizations perpendicular and parallel to the plane of incidence. The angles of incidence and refraction are θ and θ' , and n, n' are the refractive indices of the medium of incidence and the medium of transmission, respectively.



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----- **CONSTANTS** -----

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$$k=1.38\times10^{-23} \text{ J}\cdot\text{K}^{-1}$$

$$\epsilon_0=8.85\times10^{-12} \text{ F/m}$$