Advanced Optics 2008
Please start each problem on a new page. Put your BannerID on each page.
DO ANY THREE OF THE FOUR PROBLEMS.
Some of the formulas you may need may be on the equation sheet, and do not need to be rederived.

1. Near-field Diffraction.
A small round hole, radius = 200 microns, in a thin opaque screen is illuminated by plane waves of wavelength 1 micron.

a) (5 pt) Using the concept of Fresnel Zones, find how far behind the center of the hole is the farthest (local) minimum in intensity (closer than infinity)? What is the approximate intensity here, compared with the intensity if there were no screen?

b) (5 pt) Show that the intensity approximately 24 mm behind the center of the hole (on axis) is the same as if no screen were present. You should ignore the inclination factor.

c) (10 pt) Now consider that the hole is not in an opaque screen, but in a screen that reduces the intensity of light by $1/\sqrt{2}$ (everywhere except where the hole is.) What is the intensity 24 mm behind the center of the hole, compared with the incident intensity? (Hint: you may separate the light into a diffracted component and an undiffracted plane wave, and add them *appropriately* at the point of interest.)

2. Polarization

Consider a beam of light consisting of an unpolarized component and an elliptically polarized component. The polarization state of the beam is uniform across it.

Using a rotatable perfect, ideal polarizer, it is found that the maximum transmitted intensity is at 0° (or 180°) and is 4 W/cm^2.

a) (3 pt) Is there enough information to determine the angle for minimum transmission? If not, what other information do you need? If so, what angles give the minimum transmission?

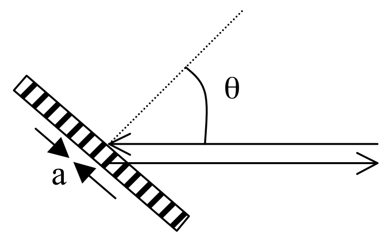
b) (3 pt) The minimum transmission is measured to be 2 W/cm^2. What is the total intensity of the original beam?

c) (6 pt) Next, a quarter wave plate is inserted before the polarizer/ analyzer, with its fast axis at 0°. The maximum transmission intensity, 5.5 W/cm^2, is now obtained with the analyzer at 30°. What intensity of the original beam is *unpolarized?*

d) (8 pt) Describe the elliptically polarized component (of the original beam): (major) axis direction, E field amplitude along major axis, the ellipticity, and the handedness.

3. **Gratings**

(a) Obtain the angle of incidence θ at which a reflection grating (grating period = *a*) retro-reflects (reflects back) the m-th order of a monochromatic incident beam at wavelength λ_0. (See figure below). *(3 points)*



(b) For a Nd:YAG laser at λ_0=1.06 μm, which of the following gratings is suitable for the above experiment: (1) 5000 lines/mm, (2) 3000 lines/mm, and (3) 500 lines/mm? Explain.
What is (are) the order (m≠0) and the incidence angle(s) at which retro-reflection occurs? *(3 points)*

(c) In part (b), what is the highest resolution (Δλ) attainable if the YAG laser beam has a diameter of 10 mm and the grating is 30 mm square? *(3 points)*

(d) First, describe the principle of "blazing" for a reflection grating. Second, find the blazing condition (angle) for the retro-reflecting grating of Part (a). Make a clear drawing of the grooves' orientation with respect to the light paths. (6 points)

(e) Describe a scanning monochromator that uses a flat reflection grating (use drawings). What are the other factors (in addition to part (c)) that limit the resolution of such a monochromator. (5 points)

Problem 4

Waveplate

Consider a quartz plate of 1.054 mm thickness. The optical axis of the crystal is in the plane of the plate.

Radiation at $\lambda_0 = 800$ nm is sent through that plate, with its (linear) polarization oriented at 45° with respect to the optics axis.

The k vector magnitudes for quartz and their derivatives at the frequency corresponding to 800 nm are:

	k μm^{-1}	$\frac{dk}{d\Omega}$ s μm^{-1}	$\frac{d^2k}{d\Omega^2}$ $\text{s}^2\mu\text{m}^{-1}$
extraordinary	12.15	$5.21 \cdot 10^{-15}$	$4.21 \cdot 10^{-32}$
ordinary	12.08	$5.18 \cdot 10^{-15}$	$4.13 \cdot 10^{-32}$

A. 4 points What is the state of polarization of radiation at 800 nm after transmission?

Consider next that the incident light has a spectrum that extends over a certain bandwidth. In this bandwidth, consider two frequencies at $\Omega_{1,2} = \Omega_0 \pm \Delta\Omega$ where $\Delta\Omega = 0.0469 \cdot 10^{15}$ s^{-1}, and Ω_0 is the angular frequency corresponding to 800 nm.

B. 4 points What is the state of polarization of the transmitted radiation at each of the two frequencies Ω_1 and Ω_2?

C. 4 points What are the transit times, at the group velocity, for wave packets centered at each of the two frequencies Ω_1 and Ω_2, for an input radiation polarized along the optic axis (e polarization)? Find the difference between these two transit times for both "e" and "o" polarizations.

Let us consider next that this spectrum is that of a Gaussian pulse (bandwidth limited), centered at 800 nm, with the electric field amplitude given by

$$\mathcal{E}(t) = \mathcal{E}_0 e^{-(\frac{t}{\tau_G})^2}.$$

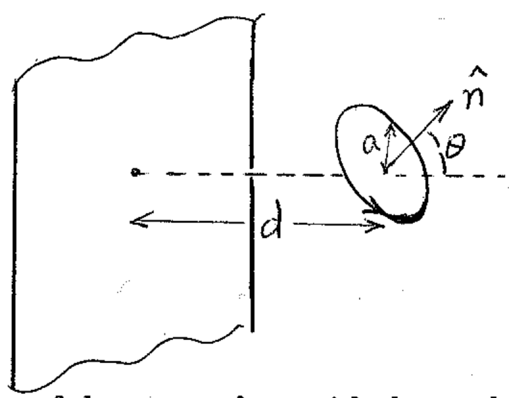
D. 4 points Knowing that $\Omega_2 - \Omega_1 = 2\Delta\Omega$ represents the Full Width at Half Maximum of the spectral intensity of the incident pulse, find τ_G.

E. 4 points In view of the above results, describe the transmitted pulse with a sketch and a few sentences. From (C), explain a change in pulse temporal behavior (output pulse width, as compared to the input pulse width). From (A) and (B), describe the evolution of the output pulse polarization versus time.

CLASSICAL E&M 2008

Please start each problem on a new page. Put your BannerID on each page. Attempt any 3 of the 7 problems here. All problems carry equal weight. Some of the formulas you may need can be found on the equations sheet. You may use them without deriving them.

1. A small coil of radius a carrying a steady current I is brought to a distance $d >> a$ from the surface of a perfectly conducting plane. The normal to the plane of the coil, as defined by the right-hand rule applied to the circulating current, makes an angle θ with the conducting-plane normal, as shown below.



(a). Show that the interaction of the current loop with the conducting plane may be described exactly by replacing the plane by an image current loop. What are the sense and magnitude of the current in the image loop, and its location and orientation? [2 pts]

(b). Compute the torque experienced by the current loop due to the presence of the conducting plane. What will be the equilibrium orientation of the loop, if it is allowed to rotate freely about a fixed diameter that is parallel to the conducting plane? [4 pts]

(c). Use the image considerations of part (a) to determine the force experienced by a long, straight bar magnet of uniform cross-sectional area A and uniform magnetization $\vec{M}$ oriented parallel to its long surface that is brought into contact with and is perpendicular to the conducting plane. Is the force attractive or repulsive? When computing the force, it may be helpful to recall that a magnetized material may be regarded as possessing an effective volume pole density $-\vec{\nabla}\cdot\vec{M}$ and surface pole density $\vec{M}\cdot\hat{n}$. (*Hint:* The magnetic induction field at the end face(s) of a long bar magnet of magnetization density $\vec{M}'$, ignoring fringing fields, is $\mu_0\vec{M}'/2$, which is just *half* of the uniform field that exists deep in its interior.) [4 pts]

2. Consider a monochromatic electromagnetic (EM) plane wave of angular frequency ω propagating along the x direction inside an infinitely large body of water of electric permittivity ϵ. The wave is scattered by a small spherical air bubble of permittivity, $\epsilon_0 < \epsilon$, and radius, $a << \lambda_0 \equiv 2\pi c/\omega$.

(a). Show that under the action of the incident EM field, the small air bubble behaves essentially as an electric dipole radiator. By applying appropriate boundary conditions to the solutions of the Laplace equation inside and outside the air bubble, show that the oscillating electric dipole moment of the air bubble is the real part of

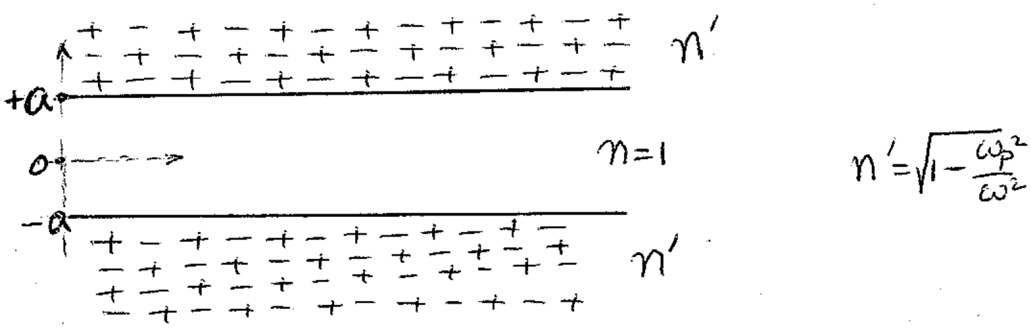
$$-4\pi\epsilon_0\left(\frac{\epsilon_r - 1}{2\epsilon_r + 1}\right)a^3\vec{E}_0\exp(-i\omega t), \quad \epsilon_r \equiv \frac{\epsilon}{\epsilon_0}$$

where the real part of $\vec{E}_0\exp(-i\omega t)$ is the incident electric field at the center of the air bubble. [5 pts]

(b). If the incident beam has a finite but large cross-sectional area, $A >> \lambda_0^2$, then what fraction of its power is scattered by the air bubble? Express your answer in terms of a, ϵ_r, ω, A, and other known/given constants. [3 pts]

(c). If there are n_0 bubbles per unit volume, each with a total scattering cross section σ, then determine the exponential attenuation of the incident wave intensity as it propagates inside the bubble-filled water body. Ignore any multiple scattering effects. [2 pts]

3. The planar surfaces of two semi-infinite plasmas bound a rectangular channel of uniform width $2a$ inside which electromagnetic waves can be confined to propagate, as shown in the figure below. Let the plasma frequency of each plasma be ω_P. Consider only TE modes in this problem.

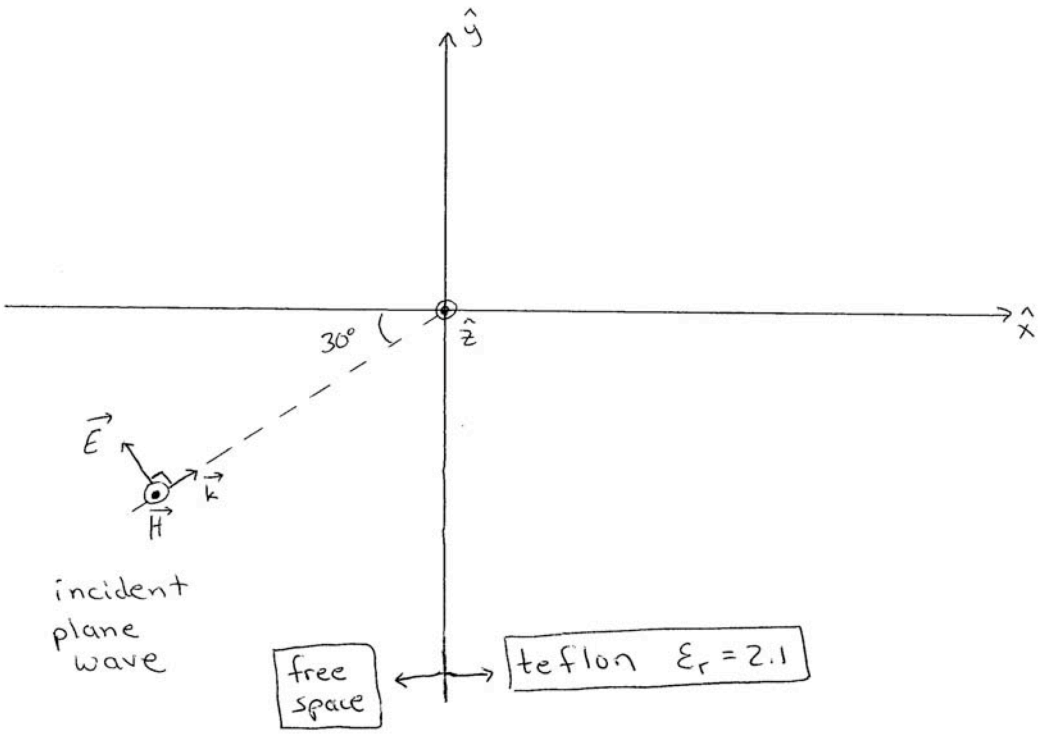


(a). Write down expressions for the electric and magnetic fields of a TE mode of frequency ω, if $\omega > \omega_P$. Use physical considerations to infer that the E field for a TE mode must be orthogonal to the propagation plane. (*Hint:* Think in terms of plane waves that are totally internally reflected at the boundaries. Also, because of the spatial symmetry along the width dimension, only solutions that are either odd or even under reflection in the center plane of the channel need be considered. The problem is analogous to the finite-well problem in QM.) [3 pts]

(b). By matching the fields at the planar boundaries of the channel via the usual EM boundary conditions, determine the eigenvalue relations that determine the two constants that represent the rate of variation of the mode fields along the thickness dimension of the channel. Consider only an even-parity mode here. [4 pts]

(c). By sketching a graphical solution, show that for $a\,\omega_P/c < \pi/2$, only a single even-parity TE mode is allowed to propagate. Determine all of the mode constants for a weakly guided TE mode in the limit $a\,\omega_P/c << 1$, and show that the wave decay constant inside the plasmas is approximately $a\,\omega_P^2/c^2$. [3 pts]

4 a) A 150 MHz sinusoidal plane wave propagates in air at 30° with respect to the +x-axis in the x-y plane. The *E*-field is polarized parallel to the plane of incidence and has a magnitude of 2 V/m. Write down the expressions for *E* of this wave in both the time-domain and the phasor domain.



b) Repeat part a) for *H*.

c) Write expressions for both the instantaneous and time-averaged Poynting vectors.

d) Now, assume that the wave considered above impinges upon a Teflon half-space located at $0 \leq x \leq \infty$ having $\varepsilon_r = 2.1$, $\mu_r = \mu_o$, and $\sigma = 0$. Write expressions for the phasor-domain reflected *E* and *H* fields.

e) Repeat part d) for the phasor-domain transmitted *E* and *H* fields.

5 a) Consider a cylindrical wave described by $A_z = A_0 H_0^{(2)}(\beta\rho)$ in free space, where $H_0^{(2)}(\beta\rho)$ is the Hankel function of the second kind of order zero and A_0 is a constant. Find the time-average Poynting vector for this wave and write it in as compact a form as possible.

b) Sketch how the time-average Poynting vector behaves as a function of $\beta\rho$.

B. CYLINDRICAL COORDINATE SYSTEM

To derive expressions for TEM modes in a cylindrical coordinate system, a procedure similar to that in the rectangular coordinate system can be used. When (6-34)

$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{A}) - \frac{1}{\varepsilon}\nabla\times\mathbf{F} \qquad (6\text{-}47)$$

and (6-35)

$$\mathbf{H} = \mathbf{H}_A + \mathbf{H}_F = \frac{1}{\mu}\nabla\times\mathbf{A} - j\omega\mathbf{F} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{F}) \qquad (6\text{-}48)$$

are expanded using

$$\mathbf{A}(\rho,\phi,z) = \hat{a}_\rho A_\rho(\rho,\phi,z) + \hat{a}_\phi A_\phi(\rho,\phi,z) + \hat{a}_z A_z(\rho,\phi,z) \qquad (6\text{-}49a)$$

$$\mathbf{F}(\rho,\phi,z) = \hat{a}_\rho F_\rho(\rho,\phi,z) + \hat{a}_\phi F_\phi(\rho,\phi,z) + \hat{a}_z F_z(\rho,\phi,z) \qquad (6\text{-}49b)$$

as solutions, they can be written as

$$\begin{aligned}\mathbf{E} = {} & \hat{a}_\rho\left\{-j\omega A_\rho - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial\rho}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right] - \frac{1}{\varepsilon}\left(\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right)\right\} \\ & + \hat{a}_\phi\left\{-j\omega A_\phi - j\frac{1}{\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial}{\partial\phi}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right] - \frac{1}{\varepsilon}\left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right)\right\} \\ & + \hat{a}_z\left\{-j\omega A_z - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right] - \frac{1}{\varepsilon}\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho F_\phi) - \frac{\partial F_\rho}{\partial\phi}\right]\right\}\end{aligned} \qquad (6\text{-}50)$$

$$\begin{aligned}\mathbf{H} = {} & \hat{a}_\rho\left\{-j\omega F_\rho - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial\rho}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}\right] + \frac{1}{\mu}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)\right\} \\ & + \hat{a}_\phi\left\{-j\omega F_\phi - j\frac{1}{\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial}{\partial\phi}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}\right] + \frac{1}{\mu}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\right\} \\ & + \hat{a}_z\left\{-j\omega F_z - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}\right] + \frac{1}{\mu}\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]\right\}\end{aligned} \qquad (6\text{-}51)$$

(The next page provides some information that you might find useful.)

Some information that you might find useful:

A derivative can be taken using either

$$\frac{d}{dx}\left[Z_p(\alpha x)\right] = \alpha Z_{p-1}(\alpha x) - \frac{p}{x} Z_p(\alpha x) \tag{IV-18}$$

or

$$\frac{d}{dx}\left[Z_p(\alpha x)\right] = -\alpha Z_{p+1}(\alpha x) + \frac{p}{x} Z_p(\alpha x) \tag{IV-19}$$

where Z_p can be a Bessel function (J_p, Y_p) or a Hankel function ($H_p^{(1)}$ or $H_p^{(2)}$).

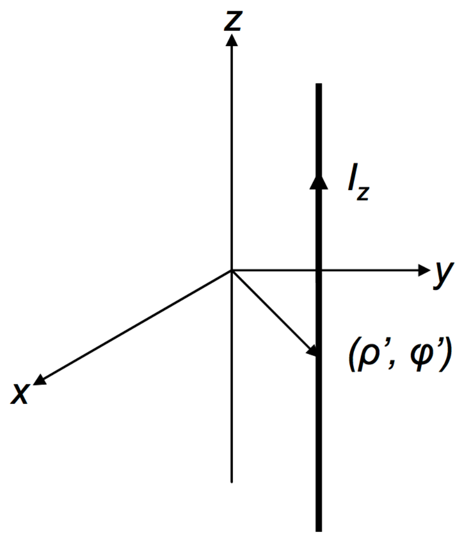
A useful identity relating Bessel functions and their derivatives is given by

$$J_p(x)Y_p'(x) - Y_p(x)J_p'(x) = \frac{2}{\pi x} \tag{IV-20}$$

and it is referred to as the Wronskian. The prime (′) indicates a derivative. Also

$$J_p(x)J_{-p}'(x) - J_{-p}(x)J_p'(x) = -\frac{2}{\pi x}\sin(p\pi) \tag{IV-21}$$

6. Consider an infinite electric line source of constant current I_z located at $\rho = \rho'$, $\phi = \phi'$ as shown below. Assume that the line source radiates in unbounded free space. Derive its Green's function $G(\rho,\phi;\rho',\phi')$.



USEFUL HINTS:

(1) The required Green's function will take the form $G(\rho,\phi;\rho'\phi') = \sum_{m=-\infty}^{\infty} g_m(\rho;\rho',\phi')e^{jm\phi}$ with

$g_m^{(1)} = A_m J_m(\beta_0\rho) + B_m Y_m(\beta_0\rho)$ for $\rho < \rho'$, and
$g_m^{(2)} = C_m H_m^{(1)}(\beta_0\rho) + D_m H_m^{(2)}(\beta_0\rho)$ for $\rho \geq \rho'$.

(2) Recall that the Wronskian of y_1 and y_2 at $x = x'$ is given by:
$W(x') = y_1(x')y_2'(x') - y_2(x')y_1'(x')$.

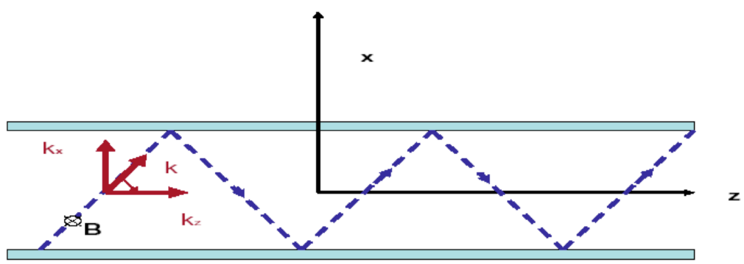
(3) The Wronskian of Bessel functions is:
$$J_n(\alpha\rho)Y_n'(\alpha\rho) - Y_n(\alpha\rho)J_n'(\alpha\rho) = \frac{2}{\pi\alpha\rho}$$

(4) The addition theorem for Hankel functions is:
$$H_0^{(1,2)}(\beta|\rho - \rho'|) = \sum_{n=-\infty}^{\infty} J_n(\beta\rho)H_n^{(1,2)}(\beta\rho')e^{jn(\phi-\phi')} \quad \text{for } \rho < \rho', \text{ and}$$
$$H_0^{(1,2)}(\beta|\rho - \rho'|) = \sum_{n=-\infty}^{\infty} J_n(\beta\rho')H_n^{(1,2)}(\beta\rho)e^{jn(\phi-\phi')} \quad \text{for } \rho \geq \rho'.$$

7. A parallel plate transmission line has a plate separation of d=1cm and is filled with Teflon having a dielectric constant of $\varepsilon = 2.1\varepsilon_0$.

(a) Find the range of frequencies over which the TE1 and TM1 (m=1) modes will propagate and no other higher modes.

(b) At an operating wavelength of λ=2mm, how many waveguide modes will propagate.



General Optics- 2008
Please begin each problem on a separate sheet. Put your BannerID on each page. Attempt all problems. Some of the equations you need may be on the equation sheet, and do not need to be rederived.

1. Consider an assembly of atoms that have energy levels separated by an energy corresponding to a wavelength of 500 nm. In thermal equilibrium, what is the ratio of the population densities in these energy levels at room temperature and at the temperature of liquid Helium? (6 pt)

2. Consider two identical lasers with a beam waist of 1 mm at the outcoupler and emitting light at wavelength λ. A pinhole of 100 micron diameter is centered on the beam just after the outcoupler (outside the cavity) of one of the lasers. Estimate the distance from the outcoupler to a screen at which both laser beams have approximately the same diameter. (6 pt)

3. Describe the operational principle of a Q-switched solid state laser. Sketch a graph that schematically shows as a function of time (use the same time axis) the (a) laser Q, (b) the population inversion, (c) the laser output power, and (d) the emitted energy. Label and explain important points on your graphs. For example, mark the population inversion at laser threshold. (14 pt)

4. A modelocked laser emits a continuous train of 1 ps pulses 10 ns apart. Estimate the number of longitudinal modes that oscillate. (6 pt)

5. Consider a coherent monochromatic beam.
   (a) Assume the beam is linearly polarized in horizontal direction. How can the polarization direction be rotated by 20 degrees? Explain how the optical component that you want to use works. (7 pt)

   (b) The beam is elliptically polarized with the major axis in horizontal direction. What can be done to flip the ellipse 90^o without changing its aspect ratio? How would you produce circularly polarized light? (7 pt)

6. (a) Fresnel (near-field) and Fraunhofer (far-field) diffraction theories are based on two different assumptions regarding the variation in the phase of light (at the observation point) originating from different points of the aperture. What are those assumptions? (3 pt)

   (b) If one wishes to calculate the diffracted intensity 10 cm past a 0.1 mm diameter hole illuminated with plane waves of wavelength 1 micron, is far-field diffraction theory appropriate? Show work (i.e. don't guess.) (3 pt)

   (c) Does your answer to (b) change if the wavelength is 1 mm? (3 pt)

   (d) Does your answer to (b) change if the light is not a plane wave, but emitted from a point source 1 mm in front of the aperture? (3 pt)

7. Describe a Fresnel rhomb. What is its use? (6 pt)

8. A collimated beam of light hits a glass surface (n=1.5) at Brewster's angle, from air (n=1.) Assume the beam has a cross-sectional area A_0 (measured perpendicular to the direction of propagation, of course.)

   (a) What is the value of Brewster's angle? (2 pt)

   (b) What polarization must the light have if there is no reflection (E parallel to the plane of incidence, or E perpendicular to the plane of incidence) ? (3 pt)

   (c) What is the area of the surface that is illuminated? (3 pt)

   (d) What is the cross-sectional area of the transmitted beam (in the glass)? (3 pt)

   (e) What is the intensity (or irradiance) of the transmitted beam, relative the incident beam? (3 pt)

9. An object 3 cm high is placed 20 cm from (a) a convex mirror and (b) a concave mirror each of focal length = 10 cm. Determine the position and nature (inverted/upright) of the image in each case. (8 pt)

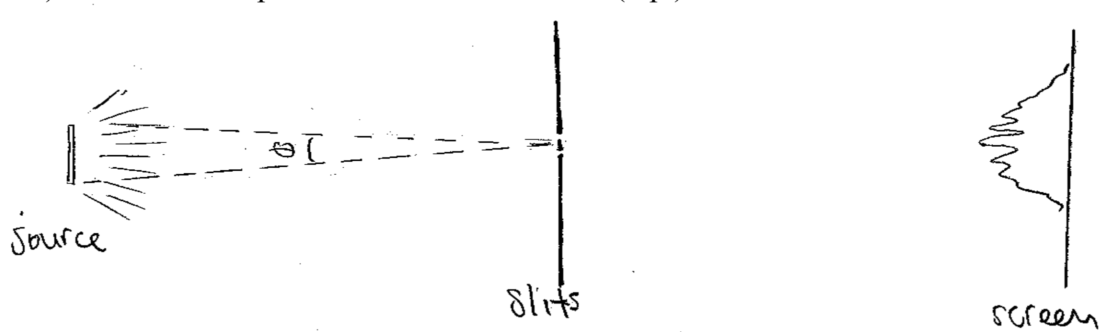
10. On the following page, the dotted line on the top graph shows the single slit diffraction pattern, using an extended, quasimonochromatic source with $\lambda \cong 500$ nm. The source is a short line segment perpendicular to the slits. The solid line shows the two-slit interference pattern, from slits of the same width, with a separation of 0.1 mm.

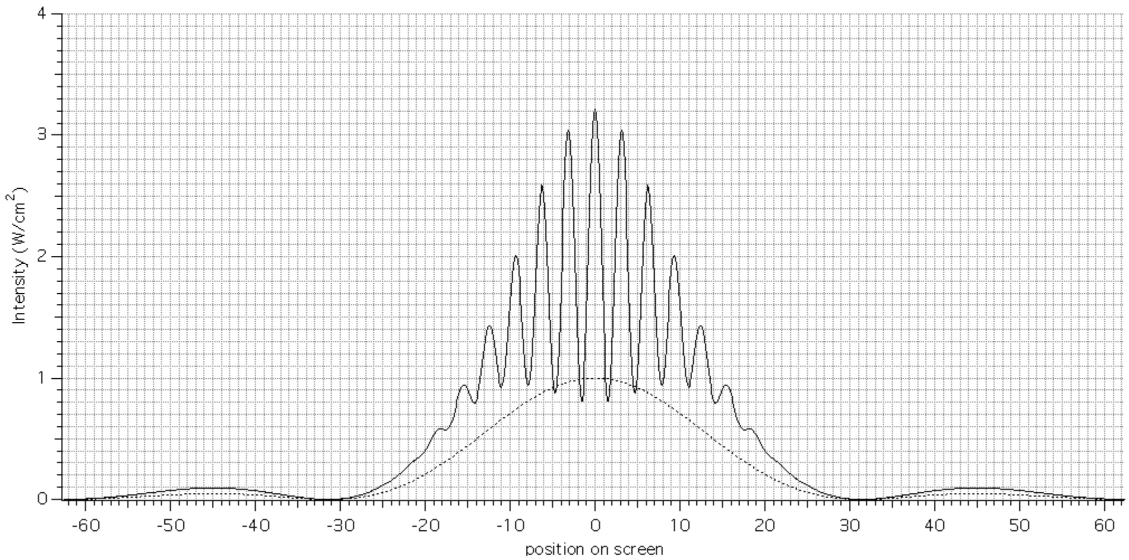
a) Sketch on the middle graph the new interference pattern if the slits are made half as wide, but kept at the same separation. Be as quantitative as you can, both in terms of x and intensity. (4 pt)

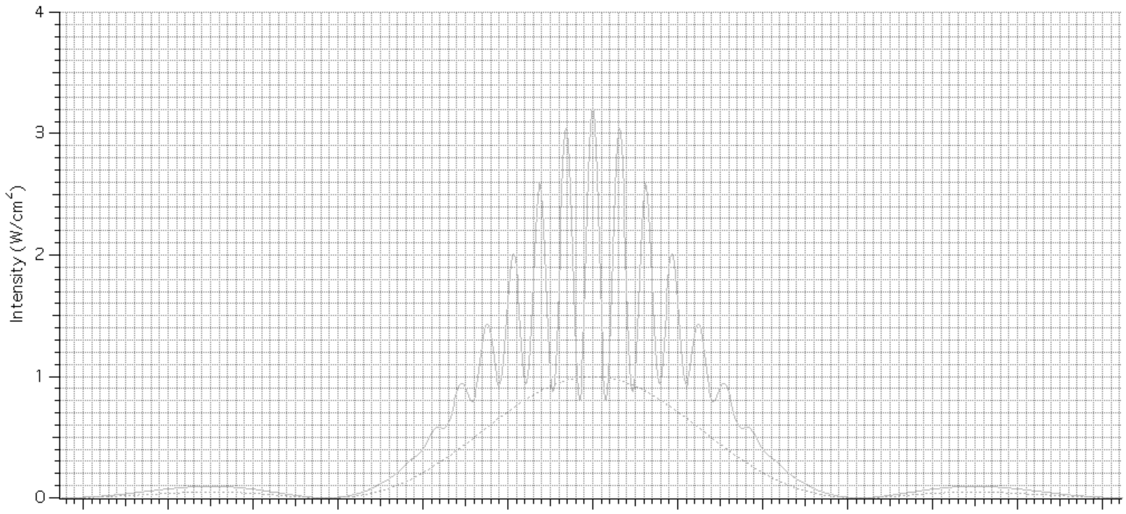
b) Sketch on the bottom graph the new pattern if the slits are kept the same, but the spectral width of the source is doubled. (4 pt)

c) If the slit separation is doubled, the interference pattern disappears. (It is visible for all smaller separations.) Estimate the angular width of the source, as viewed from the slits. (3 pt)

d) Estimate the spectral width of the source. (3 pt)




Intensity (W/cm²)
0
1
2
3
4
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
position on screen


Intensity (W/cm²)
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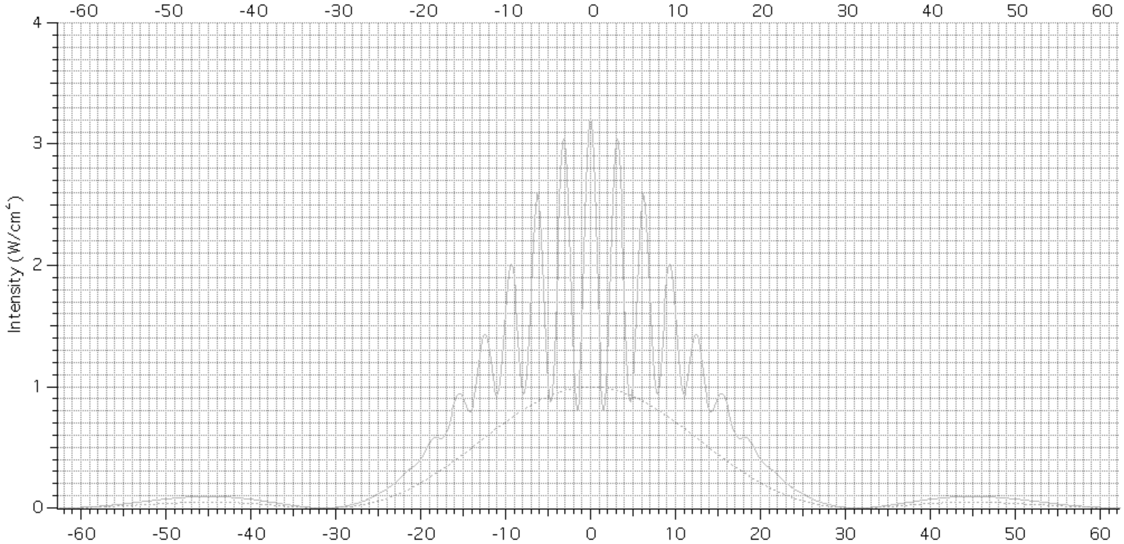
Intensity (W/cm²)
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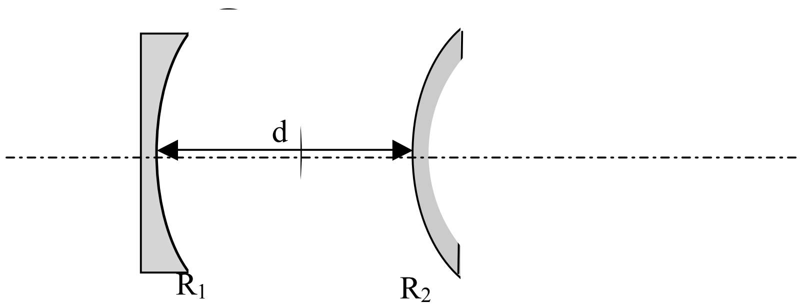
30
40
50
60

Lasers and Laser Optics – 2008
Please begin each problem on a separate sheet of paper. Put your Banner ID on each sheet. Do all problems. Some of the equations you need may be on the equation sheet, and do not need to be rederived.

Question 1.

Consider the cavity shown below consisting of two curved mirror separated by a distance d. Assume $|R_1|= |R_2|= 50$ cm, and $\lambda=1$ µm.



a) Find the range ($d_1<d<d_2$) for which the cavity is stable. (3 points)

b) Choose d=30 cm and find the location and magnitude of the beam waist (which may or may not be inside the cavity) for $\lambda=1$ µm. Qualitatively indicate your answer on the above diagram as well. (3 points).

c) Derive an expression for the resonant frequencies ($\nu_{l,m,n}$) corresponding to TEM_{mn} for this cavity (l is the longitudinal mode index). Calculate $\Delta\nu=|\nu_{l,0,0}-\nu_{l,1,1}|$. (5 points)

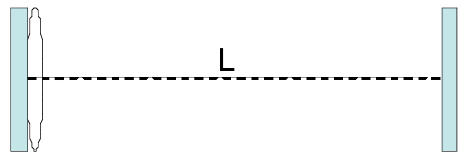
d) An external laser beam is to be coupled (matched) into this cavity. This is done by matching the **q**-parameter (i.e. spot size and curvature) of the incident beam with that of the cavity. If the incident beam is coupled from the left (i.e. mirror 1), what should be the incident beam spot size (w) and curvature (R) at the flat surface of mirror 1. (Assume thin mirror substrates with index n=1.5). (5 points).

e) Describe both active and passive modelocking and give examples for each technique. (4 points)

Question 2.

(a) A laser resonator consists of two plane mirrors and one thin lens with focal length f = 10 m in front of one of the mirrors. What is the radius of curvature of the mirror that is equivalent to the lens – plane mirror sequence in the cavity? Calculate the position and size of the beam waist if the resonator length L = 2 m and the laser wavelength λ = 500 nm.

(8 points)



(b) A thin active medium (n = 1) is inserted at the position of the beam waist and the pumping mechanism is turned on. The emitted beam has exactly the same beam parameters as one would expect from the cold (without active medium) resonator. Explain why a pump rate with a nonuniform radial distribution is necessary for this to happen.

(2 points)

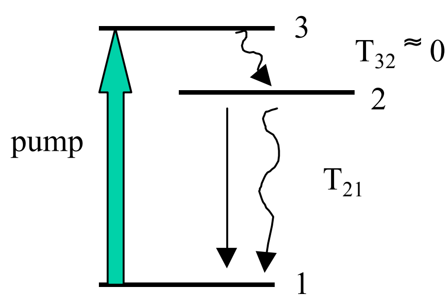
(c)

(c1) Write down the rate equations for the population number densities N_i [cm^{-3}] under lasing conditions assuming a certain pump rate R_p [$s^{-1}cm^{-3}$] and total number density N_t. For the active medium assume a three-level system, see diagram below. The homogeneously broadened laser transition is from level 2 to level 1 with cross section σ_{21} and is centered at frequency ν. For steady state, calculate the population inversion density $\Delta N_L = (N_2 - N_1)$. Neglect any dependence on the transverse (r) coordinate and ignore degeneracy factors.

(6 points)

(c2) Calculate the population inversion density ΔN_0 assuming there is no lasing. Sketch a graph showing $\Delta N_L/\Delta N_0$ as a function of the laser intensity.

(4 points)



Problem 3

A gas laser operates at a pressure of 25 torr, sufficiently high such that the gain line can be considered to be homogeneously broadened. The mirror to mirror distance is one meter, uniformly filled with the gain medium. One mirror is totally reflecting; the other one is an output coupler of reflectivity R. We will consider the gain coefficient to be uniform over the laser length ℓ, and the gain to be exponential over the round-trip length $2\ell = 2$ m of the laser. With an output coupling $R = 90\%$, the output intensity is 5 W/cm^2. For the same condition of excitation, with an output coupling of $R = 95\%$, the output intensity is 18 W/cm^2.

A. 3 points Knowing that the gain bandwidth covers 4 longitudinal modes, how many modes will oscillate? Why is this broadening mechanism called "homogeneous"?

A. 4 points List and explain all physical mechanisms of line broadening of this laser. Which mechanism dominates at high pressure, and why? Which mechanism(s) dominate(s) at low pressure?

C. 2 points What is the intracavity intensity in either case of reflection coefficient.

D. 4 points Find an expression for the saturated gain coefficient in either case of reflection coefficient. Calculate the saturation intensity as well as the small signal gain.

E. 4 points Use the above results to determine the saturation intensity of this gain medium.

F. 3 points Knowing that the inversion density in the medium is $N = 10^{16}$ cm^{-3}, and that the wavelength is 10 μm, how can the results from above be used to calculate the lifetime of the transition? Hint: you might recall that an absorber atom/molecule cannot absorb more than one photon over its cross-section until it relaxes — this relates to the saturation energy density.