General Optics – Qualifying Exam 2009

Attempt any 10 of the following problems – on your first time through, skip any problem you find difficult. All problems count equally. Begin each problem on a new sheet of paper. Put the problem number and your student ID on each sheet.

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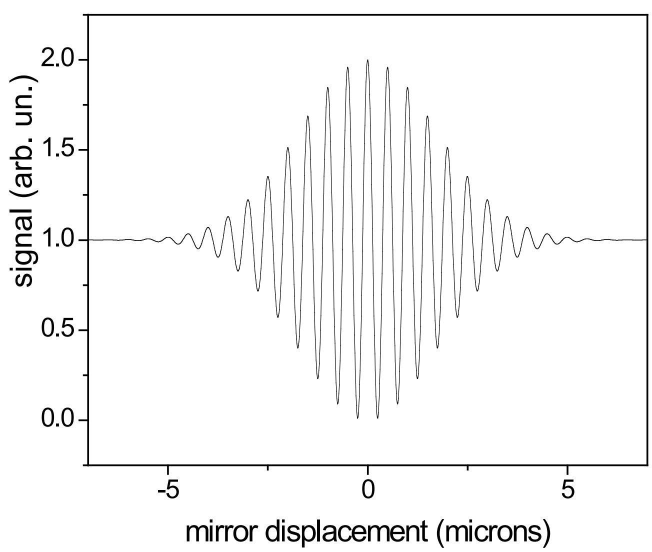
1. A detector receives one photon of green light every microsecond. What is the average power measured?

2. Consider an L = 1-m long Fabry-Perot cavity consisting of two plane mirrors that have the same (power) reflection coefficient R. Estimate R and L if the free spectral range is 10 GHz and the cavity (photon) lifetime is 100 ns.

3. An interferogram recorded with a Michelson interferometer is shown in the figure. Estimate the frequency, spectral width, and coherence time of the light source.



4. Using paraxial optics, show that two thin cylindrical lenses (focal length f) in series, aligned 90 degrees with respect to each other, are equivalent to a spherical thin lens of focal length f.

5. The US average production of electrical power is about $P_t \sim 0.5 \times 10^{12}$ W. A typical 110-V power outlet in your lab is rated for 15 A. Assume you have a laser system plugged into such an outlet that emits pulses with 10 Hz repetition rate at a center wavelength of 1 micron. The overall efficiency of your laser (average electrical to average optical power) is 10^{-4}. Estimate the pulse duration your laser must be able to produce so that the pulse power exceeds P_t. Assume that the laser consumes a constant electrical power at 90% of the maximum the outlet is able to deliver.

6. Explain in a few sentences the operational principle of
   (a) a lock-in amplifier,
   (b) a Pockels cell to select a single pulse from a train of pulses,
   (c) a (non-absorbing) polarizer to produce linearly polarized light from unpolarized light,
   (d) a blazed grating.

   Use sketches and equations if appropriate.

7.

a) The human eye has an angular resolution of about 3 mrad. How do you arrive at this number?

b) Design a telescope using only two elements (lenses and/or mirrors) with a tube length of 20 cm and sufficient resolution so that a hunter can resolve a bird's face (eyes 1 cm apart) at 2 km.

c) Discuss the resolution issues if the tube length is only 10 cm.

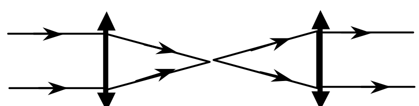
d) How can you use the telescope as a beam expander? Explain with the use of ray diagrams.

8.

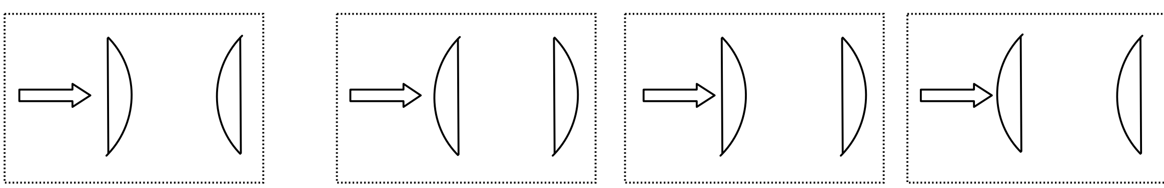
a) State Fermat's principle.

b) Use Fermat's principle to derive the law of reflection.

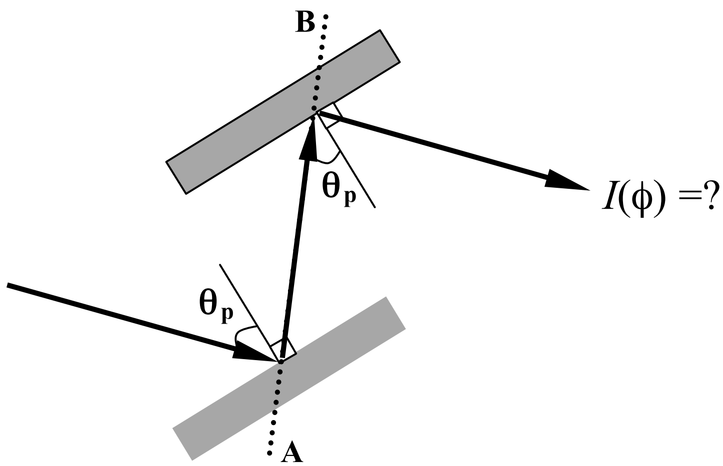
9. Collimated input light is focused and collimated again using two plano-convex lenses:



Which one of the following configurations will result in the minimum spherical aberration and why?



10. The following figure depicts a ray of light reflecting off two parallel dielectric plates at the polarization angle $\boldsymbol{\theta_p}$ (*Brewster* angle). Rotate the upper plate about the AB line through an angle ϕ so that the reflected ray comes out of the plane of the paper. Describe the irradiance of the emerging beam as a function of ϕ.

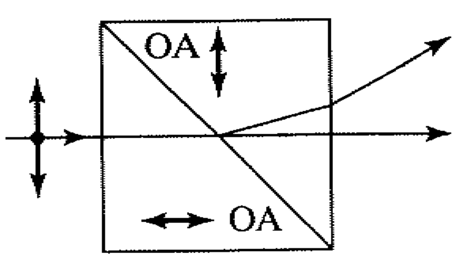


11. In 1818 Siméon Poisson deduced from Augustin Fresnel's theory the necessity of a bright spot at the centre of the shadow of a circular opaque obstacle. With his counterintuitive result Poisson hoped to disprove the wave theory; however Dominique Arago experimentally verifed the prediction.

How bright is the (brightest) Poisson spot, compared with the unobstructed incident plane wave? Show how you arrive at your answer.

12. A student attempts to view Young's double slit fringes using the sun and a monochromatizing yellow filter as a source. For what slit separations will fringes be visible? (The sun has an angular diameter of half a degree.)

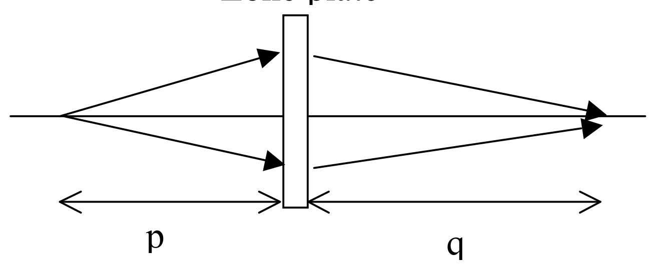
13. Unpolarized light enters a Sernamont prism as shown. Indicate on the figure the polarizations of the two emerging beams.



Advanced Optics Qualifying Examination 2009
Attempt all problems. Begin each problem on a new sheet of paper. Put the problem number and your student ID on each sheet. Staple each problem together separately. All problems count equally.

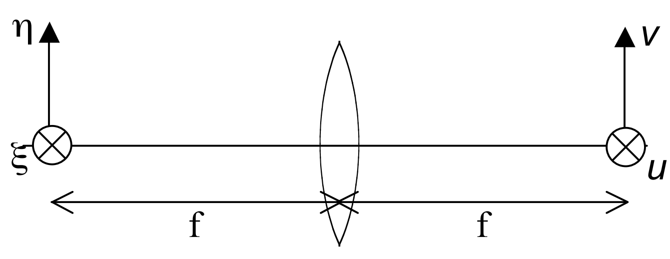
Diffraction

1-(a) Show that in a zone plate designed for focusing a spherical wave emerging from an axial point, the Fresnel zone radii should be equal to $(nL\lambda)^{1/2}$ where $1/L=1/p+1/q$, p is the distance of the source from the plate and q is the distance from the plate to the axial point of detection.



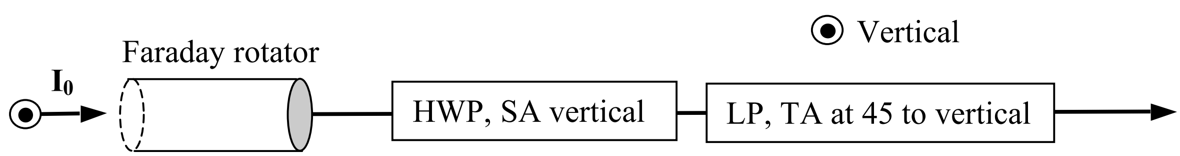
(b) A Fresnel zone plate is designed to focus the light emitted from an axial monochromatic point source 20 cm away from the plate to a point 20 cm away from the plate on the opposite side. If the same plate is illuminated by a collimated beam (same wavelength), calculate the distance between the plate and the focal point with maximum intensity.

2 - A planar input transparency with amplitude transmittance of $t_A(\xi,\eta)$ is placed in front focal plane of converging lens and is uniformly illuminated by a normally incident, monochromatic plane wave of amplitude A. Using Fresnel diffraction integral derive the diffraction pattern ($U(u,v)$) at the back focal plane and show it is exactly the Fourier transform of t_A. $U(u,v)=\mathcal{F}\{ t_A(\xi,\eta)\}$



Polarization

3-Monochromatic linearly polarized light passes through a Faraday rotator a half-wave plate (HWP) and finally a linear polarizer (LP). The original polarization and the slow axis (SA) of the HWP are vertical (perpendicular to the page) while the transmission axis of the last polarizer is 45 degree to vertical. The Faraday cell is 10 cm long with a Verdet constant of 0.0161 min/G-cm and a uniform magnetic field B is applied on the Faraday cell.



a) Show the orientation of the magnetic field for maximum rotation. What is the polarization state of the rotator output if B=10kG?
b) At what magnetic field strength the intensity of the output beam is $I_0/4$ (where I_0 is the irradiance of the input beam)?

4. The figure shows an optical system consisting of a hemispherical glass lens and a glass block. The input plane of the system is in air; the output is in glass (n=1.5), as shown.
The cardinal points are shown on the diagram.
H = principal points, N= nodal points, F = focal points.

a) Representing rays as vectors (y, α), y in cm:
what is the matrix **M** that would represent the refraction at the left (flat) surface of the hemispherical glass lens?

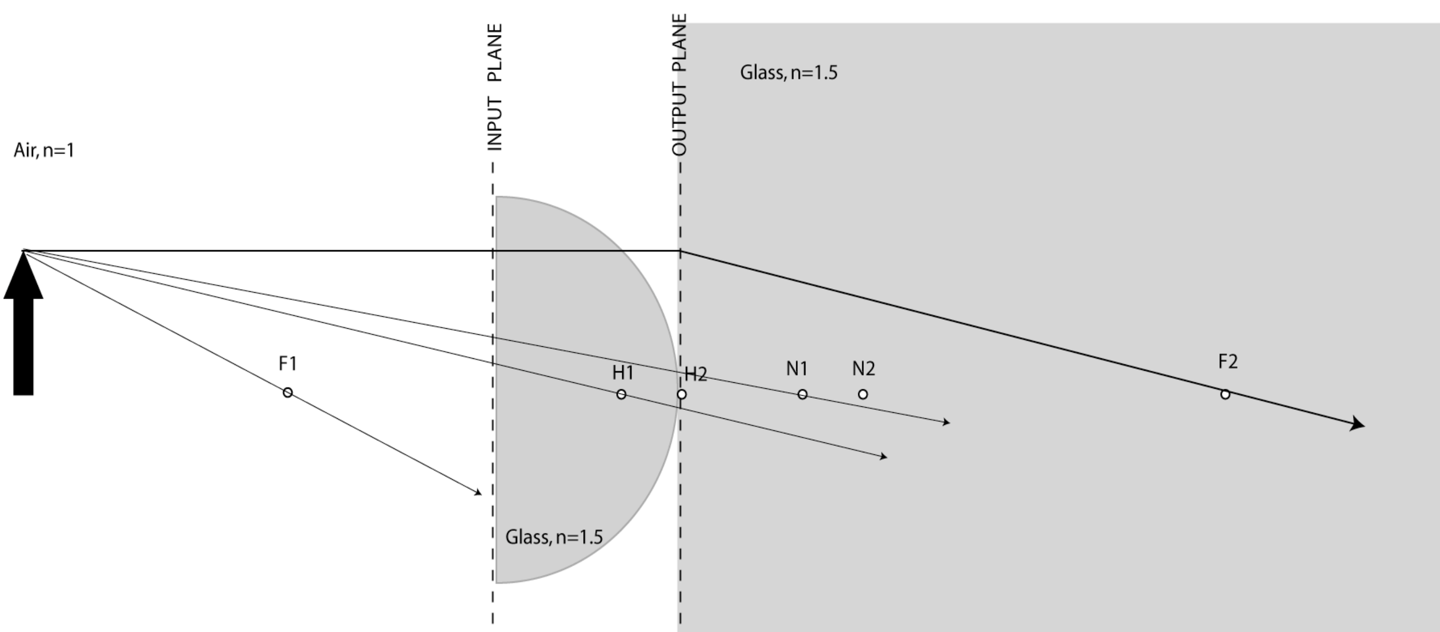
b) If **M'** is the matrix representing translation through the glass hemisphere, **M''** is the matrix representing refraction at the curved surface, and **M'''** is the matrix representing refraction at the surface of the glass block, what is the system matrix **S** ?

c) One ray emanating from the top of an object is traced for you. Trace each of the three other rays shown, *if there is an applicable ray-tracing rule for the ray.* (Treat the rays as paraxial, even though they are a bit far from the axis.)

d) The system matrix is

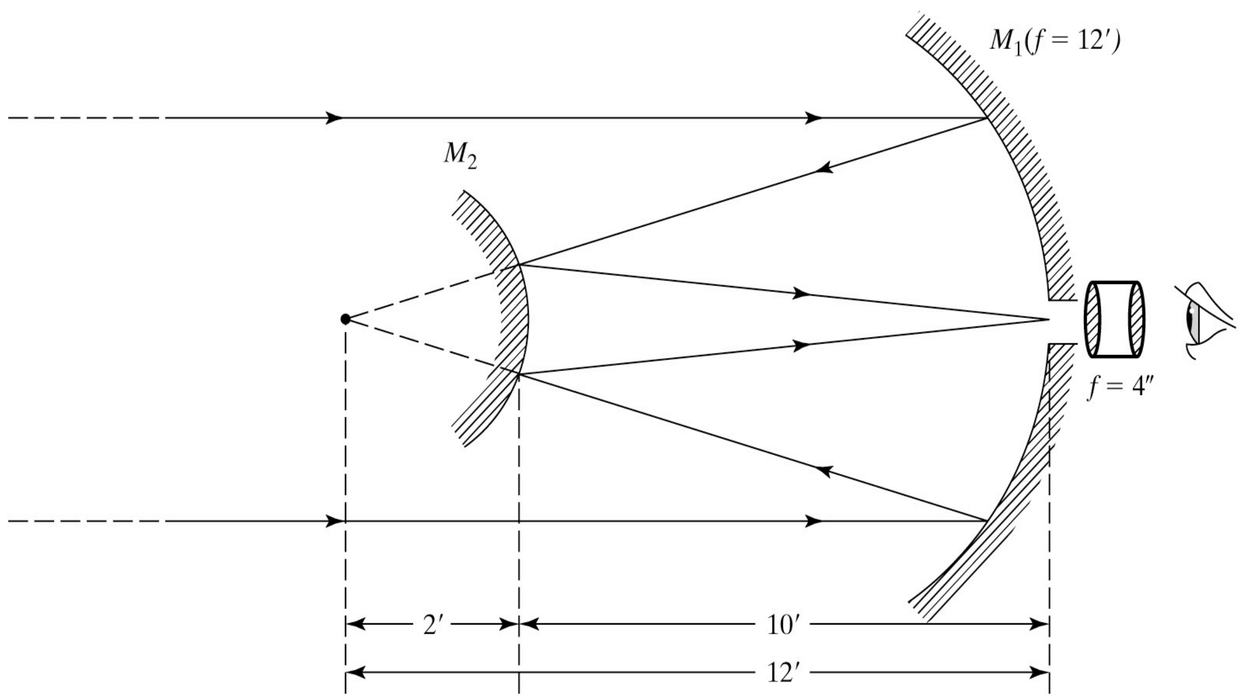
$$\begin{bmatrix} 1 & 2 \\ -1/9 & 4/9 \end{bmatrix}.$$

How far is the back (right side) focal point F2 from the output plane?
What is the back focal length?
What is the front focal length?



5.

The primary mirror of a Cassegrain reflecting telescope has a focal length of 12ft.The secondary mirror, which is convex, is 10ft from the primary mirror along the principal axis and forms an image of a distant object at the vertex of the primary mirror. A hole in the primary mirror permits viewing the image with an eyepiece of 4-in. (0.33 ft) focal length, placed just behind this mirror. Calculate the focal length of the secondary convex mirror and the angular magnification of the instrument.



Required formulas

$\beta = VBd$ Polarization rotation in a Faraday cell. *V*: Verdet constant, B; Magnetic field strength, d: Faraday cell length

$$t(x,y) = \exp\left[-i\frac{k}{2f}(x^2 + y^2)\right]$$

Phase transformation of a spherical lens

$$U_D(x,y) = \frac{ie^{-ikz}}{\lambda z}\iint_{-\infty}^{+\infty} U_A(\xi,\eta) \times \exp\left\{\frac{-ik}{2z}\left[(x-\xi)^2 + (y-\eta)^2\right]\right\} d\xi d\eta$$

Fresnel diffraction integral, z is the distance between the aperture plane (ξ,η) and the observation plane (x,y).

Jones Matrices:

TA at 45 to horizontal: $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

HWP, SA vertical: $e^{-i\frac{\pi}{2}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Qualifying Examination – Lasers – 2009
Solve ALL problems. Begin each problem on a new sheet, and staple each problem separately. All four problems count equally.

Problem 1. Consider a TEM_{00} (Gaussian) λ=500 nm laser beam propagating in free space, with a waist (w_0) of 100 μm. A screen is placed one Rayleigh range (z_0) past the waist.

We wish to align the beam from a second laser, λ=800 nm, so that it
i) propagates co-axially with the first, **and**
ii) has a waist size that is also 100 μm, **and**
iii) has the same beam radius (w) on the screen.

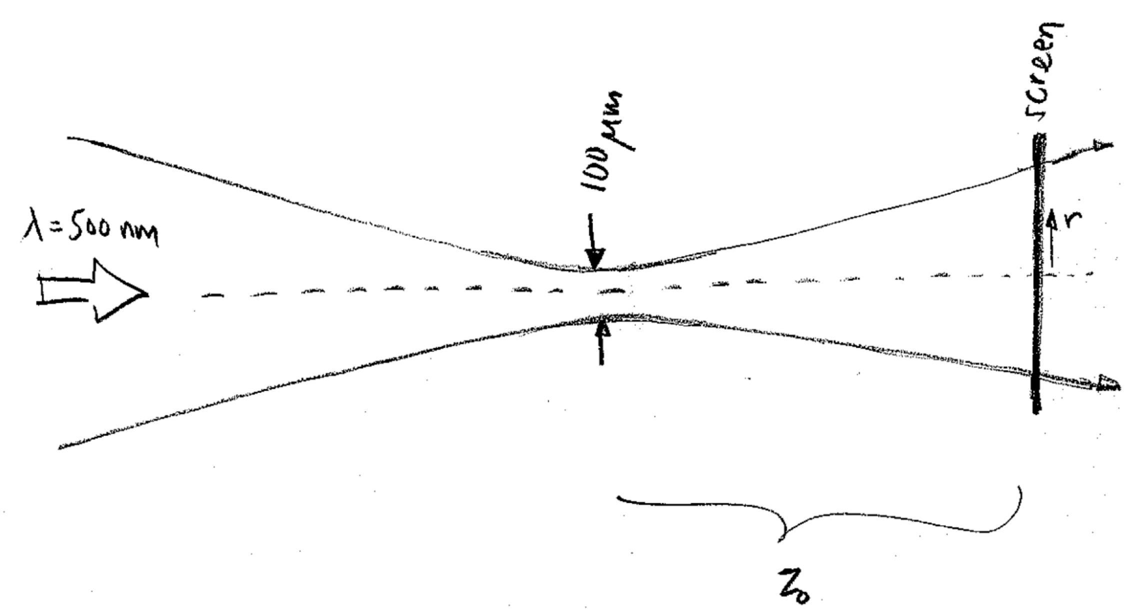
a) Is this possible? If so, where should the waist of the second laser beam be positioned?

b) What are the radii of curvature for the wavefronts from each laser, at the screen?

c) Both lasers have the same polarization. The CW 500 nm laser puts out 1 W of power. If we desire that the E-fields from each laser have the same amplitude at the screen, what must be the power of the 800 nm laser?

d) Write the E fields from each laser at the screen, as a function of the radial coordinate *r*. What is the total E field on the screen?

e) Sketch a couple wavefronts from each laser on the figure, near the screen. Is there anything remarkable about the relative phases of the fields at the position of the screen?



Problem 2

A laser beam (plane wave) is sent through a dye amplifier, with a linear gain G of 54.6 per pass. The input beam has a cross section of A = 1 mm^2, a power of P_0 = 2 kW, and a wavelength of λ = 600 nm. The length of the gain medium is ℓ = 4 cm. The density of amplifying atoms is ΔN_0 = 10^{16} cm^3. The lifetime of the gain medium is 3 ns.

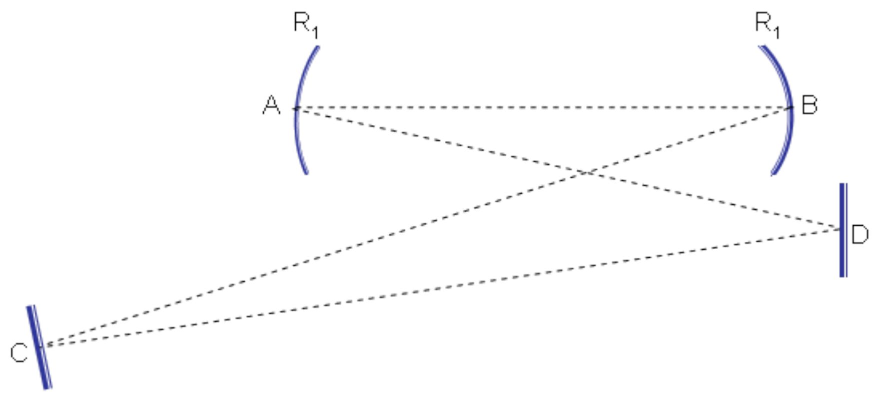
a) 2 pts Find the linear gain coefficient α_0, the amplification cross section σ, and the saturation intensity I_s for this medium.

b) 4 pts Derive an expression for the transmitted power versus input power.

c) 4 pts The cross section A is now a variable. There is a fixed pump power, which creates an inversion $\Delta N_0 = 7 \cdot 10^{14}/A$ (the larger the beam cross section, the smaller the gain, but the larger the saturation of the gain). What is the maximum power that can be extracted from this amplifier? In which limit is that maximum power extraction achieved (large cross section A or small cross section?)

Problem 3.

Consider the ring cavity shown below consisting of two curved mirrors (of same radius of curvature R_1) and two flat mirrors. Dimensions are: AB=12cm, BC=25cm, CD=25cm, and AD=20cm.



(a) How many beamwaists are in this cavity and where are the locations? (You do not need calculations to answer this question).

(b) Obtain the ABCD matrix for this cavity. Clearly state your starting point and direction of rays. Assume the angle of incidence on the curved mirrors is small and will not cause any aberrations.

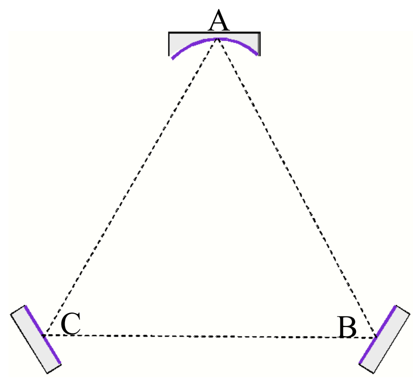
(c) Describe the procedure for calculating the Gaussian beam parameters (at any given point) in this cavity?

(d) What is the range of R_1 for which this cavity is stable?

(e) Briefly discuss the effect of oblique incidence (on the curved mirrors) on the beam parameters.

Problem 4.

Consider a unidirectional ring laser (shown below) with a cavity length of 120 cm. The cavity is filled with an inverted gas having the following characteristics: spontaneous lifetime t_{sp}= 10^{-5} s, $\Delta\nu$=1 THz (pressure broadened), laser wavelength λ=1.3 μm. The lower state depopulates very fast while the upper state lifetime is dominated by spontaneous emission. Assume n=1.
Mirror B is the output coupler with a transmission of 2% while the other mirrors are highly reflecting (R=100%). Average beam area is 1 mm^2.



(a) What is the threshold upper state population?

(b) What is the saturation intensity?

(c) What is the passive cavity photon lifetime?

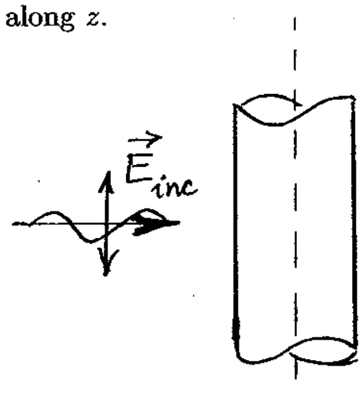
(d) If pumped 5 time above threshold, estimate the out put power.

(e) If modelocked, estimate the pulse width and peak power (pumped 5x above threshold).

CLASSICAL E&M 2009

Please start each problem on a new page. Put your BannerID on each page. <u>Attempt any 3 problems. All problems carry equal weight.</u> Some of the formulas you may need can be found on the equations sheet. You may use them without deriving them.

1. A plane EM wave of angular frequency ω is scattered by an infinitely long, perfectly conducting cylinder of radius a. The wave is incident along the x axis while the axis of the cylinder is oriented along the z axis. Let the incident wave be linearly polarized along z. One may then argue, rather simply, that the scattered field must also be linearly polarized along z.



(a). Express the x-dependent incident electric field in polar coordinates as a series of products of ordinary cylindrical Bessel functions of $k = \omega/c$ times the radial coordinate, ρ, and exponential functions in the azimuthal angle, ϕ. (*Hint:* Use the generating function for Bessel functions.) [2 pts]

(b). Express the scattered electric field as a similar series involving suitable Hankel functions, paying close attention to the outgoing-wave boundary condition for this wave. [2 pts]

(c). Apply appropriate conducting-surface boundary condition at $\rho = a$ to the full radiation field and thus determine the unknown coefficients in the series representing the scattered electric field in terms of the incident electric-field amplitude E_0 and appropriate Bessel functions. Show that in the limit $ka << 1$, the coefficient of the mth order Hankel function in this series decays as $1/\ln(2/ka)$ for $m = 0$ and as $(ka)^{2m}$, for $m \geq 1$. (*Hint:* For large arguments, $H_m^{(1)}(u) \rightarrow \sqrt{2/(\pi u)}\exp[iu - (m + 1/2)\pi/2]$, whereas for $u \to 0$, $J_m(u) \to (u/2)^m/m!$, $H_0^{(1)}(u) \to -(2i/\pi)\ln(2/u)$, and $H_m^{(0)}(u) \to -i(2/u)^m(m-1)!/\pi$ for $m = 1, 2, \ldots$.) [2 pts]

(d). Keeping only the lowest order ($m = 0$) term in the scattered electric field, derive an expression for the scattered magnetic field in the radiation zone. Using the expressions for the E and B fields in the radiation zone, determine the total time-averaged power scattered by the cylinder per unit length. (*Hint:* To compute the scattered power, use the Poynting vector at large distances from the cylinder.) [4 pts]

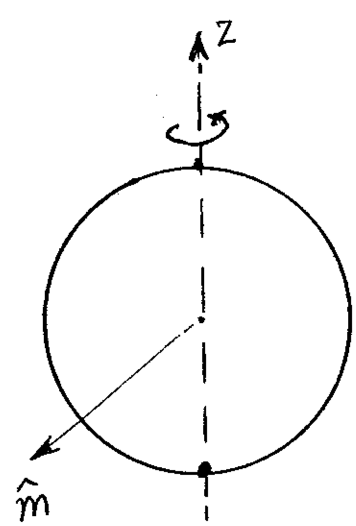
2. A plane circular coil of radius a carrying a steady current I rotates uniformly about a diameter such that its unit normal vector $\hat{m}$ has the following time-dependent form:

$$\hat{m} = \cos\omega t\ \hat{x} + \sin\omega t\ \hat{y}.$$

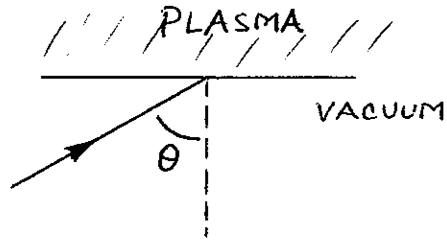
(a). Express the magnetic dipole moment vector of the current loop as a function of time. [2 pts]

(b). In the limit $\omega a/c << 2\pi$, write down the electric field of radiation at large distances from the coil along a general direction given by the unit vector $\hat{n}$. Based on this expression, determine the polarization of the EM field radiated in the direction (i) along the axis of rotation and (ii) perpendicular to the axis of rotation. How can one understand these polarization results based on simple physical considerations alone? [4 pts]

(c). Imagine that a point charge q oscillates sinusoidally along the diameter about which the coil rotates, reaching from one end of the diameter to its other end in half the period of rotation, *i.e.*, the coil-rotation and charge-oscillation periods are identical. At $t = 0$, the charge starts out at one end of the diameter. Express the field radiated by the combined system of rotating current and oscillating charge along the $\hat{x}$ direction. Show that the power radiated along x cannot vanish regardless of the charge to current ratio, q/I. [4 pts]



3. A plane wave of angular frequency ω propagating in vacuum is incident on an idealized, semi-infinitely extended plasma with plasma frequency $\omega_p = \omega/2$. At the interface of the plasma with the vacuum, the wave is incident at an angle θ with respect to the interface normal.



(a). For what incidence angles is the wave not at all transmitted into the plasma? [2 pts]

(b). Let the incidence angle θ be 75^0 and the incident wave be right circularly polarized. By deriving an expression for the reflected electric field in terms of the Fresnel reflection coefficients for linear polarizations, show that the reflected wave is in general elliptically polarized. [4 pts]

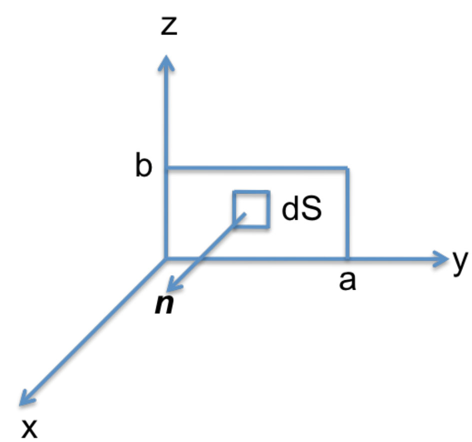
(c). Return now to the case of an arbitrary angle of incidence, θ, for which there is no transmission into the vacuum. Let ϕ be the difference of the phase shifts acquired in reflection by the two linear-polarization states denoted by real unit vectors, $\hat{e}_1$ and $\hat{e}_2$, in terms of which the incident circular-polarization (CP) state may be expressed. By re-expressing the reflected electric field in the CP basis, $\hat{\epsilon}_\pm = (1/\sqrt{2})(\hat{e}_1 \pm i\hat{e}_2)$, show that the relative phase of the coefficients of the two CP basis vectors is $\pi/2$, regardless of the value of θ. (This means, as one can show (but you need not), that the major and minor axes of the elliptical polarization of the reflected field are always oriented at 45^0 to the plane of incidence, regardless of θ.) [4 pts]

4.

The electric field of radiation inside a hollow rectangular waveguide whose geometry is sketched below, has the following form:

$$E_y = j\frac{\omega\mu_0 H_0 b}{\pi}\sin\left(\frac{\pi z}{b}\right)e^{j\omega t-\gamma x},\ E_x = E_z = 0;$$

where $\gamma = \left[\left(\frac{\pi}{b}\right)^2 - \omega^2\mu_0\varepsilon_0\right]^{1/2} > 0$ and the other symbols have their usual meaning.



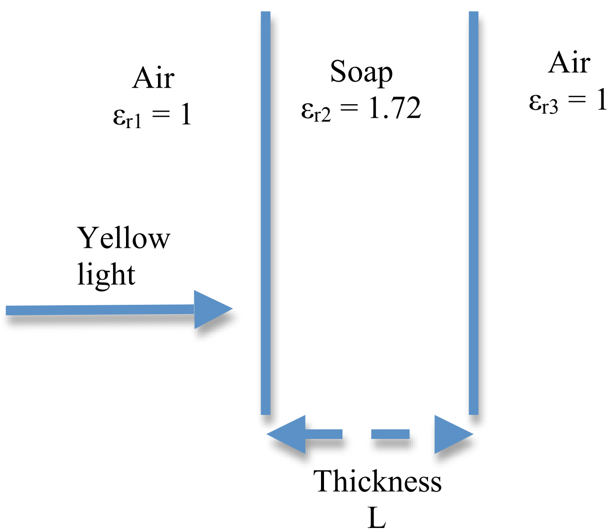
a) Derive expressions for the three Cartesian components of the magnetic field of radiation inside the guide.

b) Calculate the average power flowing across any plane transverse to the x-direction. Provide a simple physical explanation for your answer.

<u>5.</u>

Q2. a) Consider a thin film of soap in air under illumination by yellow light with $\lambda = 0.6$ μm in vacuum. If the film is treated as a planar dielectric slab with $\varepsilon_r = 1.72$, surrounded on both sides by air, what *minimum* film thickness would produce the strongest reflection of the yellow light at normal incidence? How much is reflected?

Q2. b) At what intervals of film thickness greater than the minimum found in part (a) is this strong reflection reproduced?

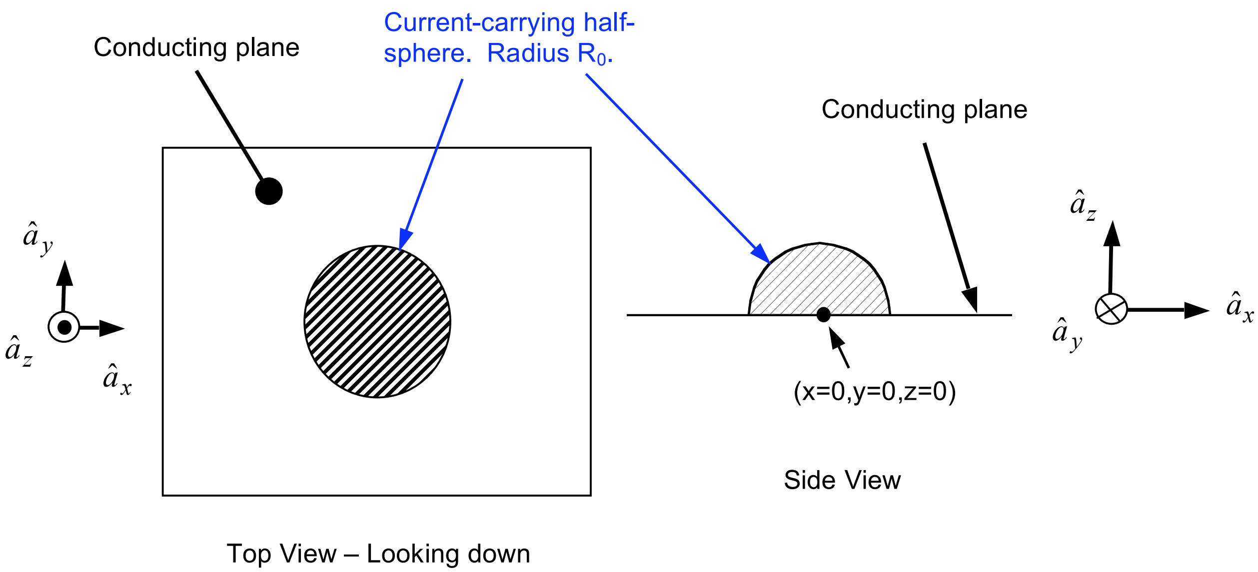


6.

Consider an oscillating surface current density, $\underline{J}_s$, flowing on a hemispherical structure located above an infinite, perfectly conducting plane (located in the x-y plane), as shown below. Assume the conducting plane is infinitely large in the x and y directions. The sphere has radius R_0, and is centered at (x,y,z) = (0,0,0).
The surface current density at r = R_0 is given by

$$\underline{J}_s = \hat{a}_\phi J_0 \sin\theta e^{j\omega t} ,$$

where (r,ϕ,θ) are the usual spherical coordinates.



a) Find the approximate radiated vector potential $\underline{A}$ in the far field in the region z > 0. Make any assumptions you think are reasonable.

b) Find the approximate radiated electric field, $\underline{E}$, in the far field.

Useful Formulas:

- Generating function for Bessel functions:

$$\exp(iz\cos\theta) = \sum_{n=-\infty}^{\infty} i^n \exp(in\theta) J_n(z).$$

- Magnetic induction $\vec{B}$ due to a static magnetic dipole $\vec{m}$:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{[3(\vec{m}\cdot\vec{n})\,\vec{n} - \vec{m}]}{r^3}, \quad \vec{n} = \frac{\vec{r}}{r}.$$

- Complex magnetic-induction field radiated by a sinusoidally varying magnetic dipole, $Re\left(\vec{m}\, e^{-i\omega t}\right)$, in the radiation zone:

$$\vec{B} = \frac{\mu_0 k^2}{4\pi}(\hat{n}\times\vec{m})\times\hat{n}\frac{e^{ikr-i\omega t}}{r}.$$

- Complex electric field radiated by a sinusoidally varying electric dipole, $Re\left(\vec{p}\, e^{-i\omega t}\right)$, in the radiation zone:

$$\vec{E} = \frac{k^2}{4\pi\epsilon_0}(\hat{n}\times\vec{p})\times\hat{n}\frac{e^{ikr-i\omega t}}{r}.$$