General Optics – 2010

Do any 10 of the 13 problems. All problems count equally. Begin each problem on a new page. Put your ID# (not name) on the top of each page. Most problems can be done in less than one page.

1. Describe Q-switching and typical characteristics of a Q-switched laser output. Give one example each for both active and passive Q-switching techniques. *Do not exceed 1 page.*

2. Describe modelocking and give typical characteristics of modelocked laser output. Give one example each for both active and passive modelocking techniques. *Do not exceed 1 page.*

3. Assuming reasonable dimensions and temperatures, estimate the power radiated by a human body. What is the *net loss* of Calories from the human body (through radiation) from standing in the cold (2 ^{o}C) naked for 1 hour (assume an ideal blackbody)? Give your answer in terms of the number of brownies: each brownie has about 100 Calories and each (food) Calorie is 4.18 kilojoules).

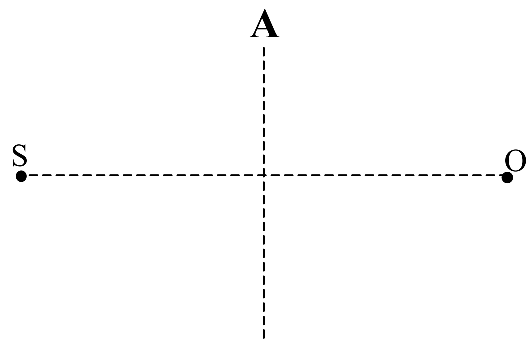
4. Give the essential characteristics of single mode and multimode fibers. Discuss the implications for long haul optical communications? *Do not exceed 1 page.*

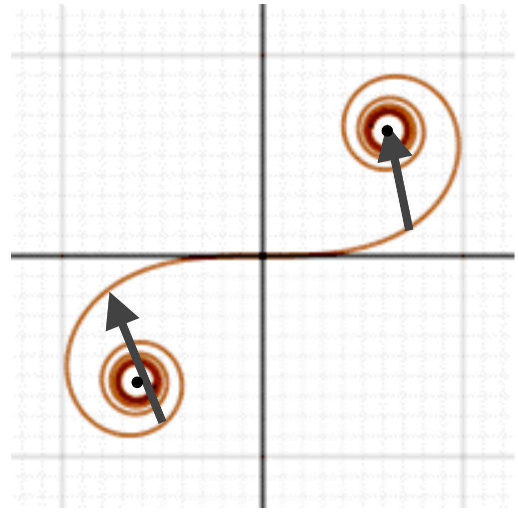
5. Describe the basic principles of an LCD display (such as a computer laptop screen). *Do not exceed 1 page.*

6. An astronaut is looking at point source in a training pool (filled with water, n=1.33). If the point source is 100 cm from the vertex of the transparent hemispherical shield in front of the helmet, find how much and in what direction (closer or farther) the observed image is displaced relative to the actual point source (Radius of curvature of the shield = 20 cm; you may assume the shield is infinitely thin.)

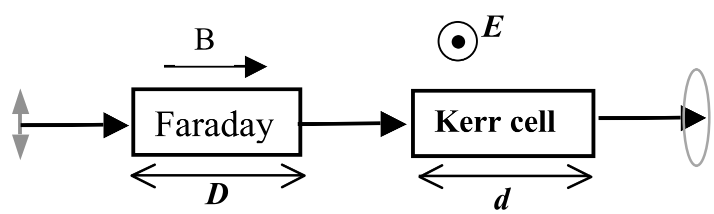
7. A binary star system in the constellation Orion has an angular interstellar separation of 10^{-5} rad. If λ = 500nm, what is the smallest diameter the telescope can have to just resolve the two stars?

8. A coherent line source S (perpendicular to the page) is placed behind an aperture screen (A). The vectors on the Cornu spiral represent the diffracted optical fields at point O. Roughly sketch the location and the width of the slit(s) in the aperture screen.

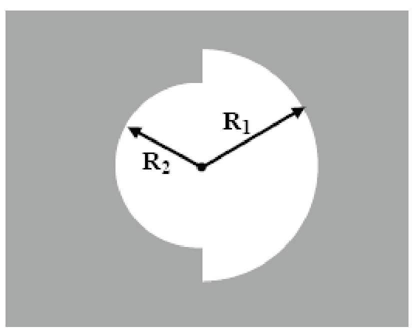




9. A Faraday and a Kerr modulator are to be used to transform a linearly polarized beam to a circularly polarized beam. Find the relation between the electric field and the magnetic field applied to the modulators (as shown), in terms of the Verdet constant V, Kerr constant K, and the lengths of the modulators D and d.



10. A plane wave with a wavelength of λ = 500 nm illuminates an opaque screen having an aperture shown below (where R_1=2 mm and R_2=1.414 mm):



Estimate the irradiance at an axial point 4 m from the screen.

11. A Gaussian TEM_{00} beam coming out of a beam expander is used to illuminate a target at a distance L= 500 m. The beam has a waist at the output of the expander.
If the beam has a wavelength of 500 nm and total power of 1 Watt: (a) find the waist (at the expander output) that guarantees the highest peak intensity on the target. (b) Calculate this intensity.

12. An image of an object is formed on a screen by a lens. Leaving the lens fixed, the object is moved to a new position and the screen is moved until again a sharp image appears. If the two object positions are s_1 and s_2 and if the transverse magnification of the two images is m_1 and m_2, respectively, show that the focal length of the lens is given by

$$f = \frac{s_1 - s_2}{\frac{1}{m_1} - \frac{1}{m_2}}$$

13. Linearly polarized light (a plane wave of irradiance I_0) is incident on a sequence of two crossed polarizers, see figure. The pass direction of the first polarizer, P1, is at an angle of $\beta = 22.5°$ with respect to the polarization of the incident light.

a) You have two quarter wave plates (QWPs). Where should you place them and what should be their orientation to maximize the irradiance at the detector?

b) Now, in addition to the QWPs, you have one polarizer at your disposal. Where should you place the QWPs and the polarizer to obtain maximum irradiance at the detector? What is the maximum irradiance at the detector plane, in terms of I_0 ?

Advanced Optics 2010

Answer all 5 questions. Each counts equally. Begin each question on a new sheet of paper, put your ID# (not your name) at the top of each page. Staple all pages of each problem together, but do not staple your entire exam together.

1-(a) Derive the relation between blaze angle, diffraction order and grating period for a Littrow configuration (blazed grating where light is normally incident to the groove face). (b) Determine the blaze angle for a 4-cm long Littrow mount with 1000 grooves to maximize the reflected energy at λ=500 nm in the 3^{nd} order. (c) Estimate the resolving power of the grating in the same order.

2-A double-slit aperture is illuminated by a coherent planar wavefront with λ = 600 nm. The width of each slit is w, the distance between the slits (center-to-center) is a = 0.6 mm. There is a screen a distance L =1 m (>> a^2/λ) past the slits.

(a) If the sixth peak (after the central peak) in the fringe pattern is missing, what is the width of each slit?
(b) Sketch the fringe pattern observed on the screen up to the 12^{th} fringe.
(c) If the point source has a finite coherence time of τ = 1 ps, how far from the center of the screen does the fringe visibility drop to 50%?
(d) If we replace the plane wave with a transversely extended incoherent source at a large distance L' (L' >> a) in front of the slits, estimate the characteristic maximum source size that results in a non-zero fringe visibility.

3 – Newton's (interference) rings are formed between a spherical lens surface and an optical flat. If the 10^{th} bright ring of green light (520 nm) is 8 mm in diameter, what is the radius of curvature of the lens surface?

4 – The indices of refraction for the fast and slow axes of quartz (with 546 nm light) are 1.5462 and 1.5553, respectively.

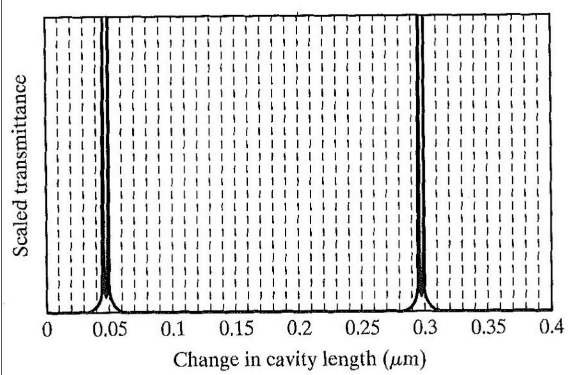
(a) What is the thickness of a zero-order quarter-wave plate (QWP)?
(b) Estimate the bandwidth, $\Delta\lambda$, of light centered at 546 nm this zero-order QWP can handle, if the phase retardation must not deviate more than 5% of the desired value. You may assume the refractive indices are non-dispersive.
(c) Repeat the estimation for a QWP of order m.

5 – Consider a light source consisting of two monochromatic components with different wavelengths λ_1 and λ_2 . Let light from this source be normally incident on a scanning Fabry-Perot interferometer (FPI) of nominal length d=5 cm. The transmittance through the FPI as a function of change in the cavity length is shown in figure (a). Figure (b) is an expanded version showing only the first two transmission peaks.

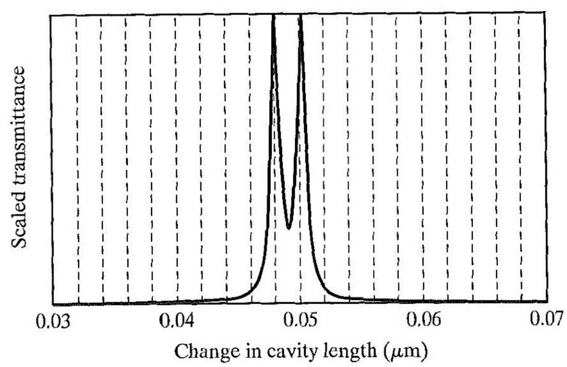
(a) What is the nominal (average) wavelength of the light source?

(b) Estimate the difference λ_1 - λ_2 in wavelength of the two spectral components, assuming that the overlapping transmittance peaks belong to the same interference order m.

(c) Estimate the difference λ_1 - λ_2 in wavelength of the two spectral components, assuming that the overlapping transmittance peaks belong to successive interference orders.



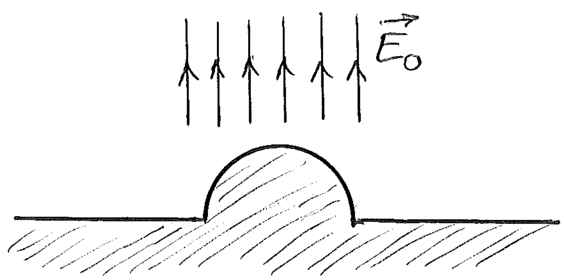
(a)



(b)

Electromagnetics -2010. Answer any 3 of the 7 questions. Put your ID# (not your name) at the top of each page. Begin each question on a new page. Staple all pages for each problem together, but do not staple your entire exam together.

1. The surface of an infinitely extended, grounded conductor is planar except for a hemispherical bump of radius a. The conductor is placed in a uniform external electric field $\vec{E}_0$ that is normal to the planar surface of the conductor.



   (a) Write down the electric potential of the external field in the absence of the conductor in a suitably chosen spherical polar coordinate system.

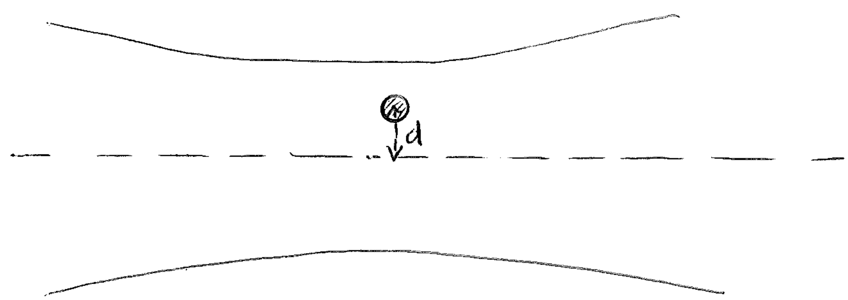
   (b) By adding an appropriate potential to that of part (a) and then applying an appropriate boundary condition, solve for the potential everywhere outside the conductor.

   (c) Show that the electric field on the bump decreases as the boundary between it and the planar surface is approached. What is its value at that boundary? Explain this behavior in physical terms.

2. A small dielectric sphere of dielectric permittivity ϵ and radius a is placed in the waist of a focused, monochromatic, linearly polarized radiation field of wavelength λ that is large compared to a. Let the radiation beam intensity have the form,

$$I(\vec{\rho}) = I_0 \exp(-\alpha\rho^2),$$

where α is a positive constant and ρ is the radial coordinate transverse to the direction of beam propagation. Let the center of the sphere be a distance d from the axis of the beam ($\rho = 0$) in the beam waist.



(a) What is the time-dependent electric field of radiation at a distance ρ from the beam axis? Express your answer in terms of I_0, ρ, λ, and familiar electromagnetic constants. Allow for an arbitrary phase constant in your answer.

(b) By invoking the long-wavelength approximation, write down the induced electric dipole moment of the sphere in terms of the electric field of radiation and ϵ. (*Hint:* The electric field inside a dielectric sphere placed in a uniform electrostatic field $\vec{E}_0$ is $3\vec{E}_0/(\kappa+2)$, where $\kappa \equiv \epsilon/\epsilon_0$.)

(c) What is the time-averaged force experienced by the dielectric sphere as a function of d? Show that this force is directed toward the beam axis.

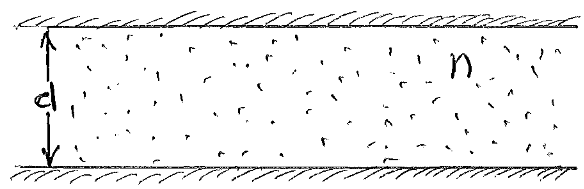
3. Two circularly polarized plane waves of opposite helicity but with the same frequency, wave vector, and amplitude are coherently superposed. Let the phase constants of the two waves be ϕ_+ and ϕ_-.

   (a) Show by using the complex notation that the resulting plane wave is linearly polarized.

   (b) How does the direction of the resulting linear polarization depend on the relative phase, $\Delta\phi \equiv \phi_+ - \phi_-$, between the two initial circular polarizations?

   (c) The two waves are now allowed to enter an optically active medium, which changes their propagation constants to $k_\pm$, where the two signs refer to the opposite helicities of the wave polarizations. What is the detailed nature of the polarization of the combined wave after a distance L of propagation in the medium? After what propagation length will the original linear polarization be restored? Express your answers in quantitative terms involving $k_\pm$ and L.

4. A slab of glass of thickness d and refractive index n is contained between two perfectly conducting planes.



(a) Argue physically why the system can serve as an electromagnetic waveguide.

(b) What types of waveguide modes are possible in such a system?

(c) By regarding the problem as that of two plane waves propagating obliquely but symmetrically upward and downward inside the slab, determine the eigenvalue condition for the existence of a TE mode of angular frequency ω. (*Hint:* The total phase shift of each plane wave in a "round trip" involves the phase shift of propagation and of two reflections, one from each bounding plane, and must be equal to an integral multiple of 2π for a sustained propagation of the waves inside the guide.)

(d) What are the cut-off frequencies of the various TE modes? Interpret your answer in physical terms.

(e) By a simple superposition of the electric and magnetic fields of the two plane waves, determine the mathematical form of the overall electric and magnetic fields of a specific TE mode.

5. Consider a *bianisotropic* medium characterized by the constitutive relations $\mathbf{D} = \varepsilon\mathbf{E} + \alpha\mathbf{H}$ and $\mathbf{B} = \beta\mathbf{E} + \mu\mathbf{H}$.

Q1.a) Show that $\mathbf{E}$ and $\mathbf{H}$ satisfy the equation $\nabla^2\mathbf{F} + j\omega(\beta - \alpha)\nabla \times \mathbf{F} + \omega^2(\varepsilon\mu - \alpha\beta)\mathbf{F} = 0$. (NOTE: $\mathbf{F}$ is <u>not</u> the electric vector potential. Substitute $\mathbf{E}$ and $\mathbf{H}$ for $\mathbf{F}$.) [6 points]

Q1.b) Show that the same equation is satisfied by the vector potential $\mathbf{A}$, provided that the gauge condition is $\nabla\cdot \mathbf{A} + j\omega(\varepsilon\mu - \alpha\beta)\phi = 0$ where ϕ is the scalar potential. [2 points]

Q1.c) Show that the corresponding scalar potential satisfies $\nabla^2\phi + \omega^2(\varepsilon\mu - \alpha\beta)\phi = 0$. [2 points]

6. An airplane is flying 1,000 m above the ocean, which has $\sigma = 4$ S/m, $\varepsilon_r = 80$, $\mu = \mu_0$. The airplane radiates 9,000 Watts at 10,000 Hz in a uniform spherical wave (i.e., the wave intensity is not a function of angle with respect to the airplane).

(a) Find E_{incident} at the surface of the ocean.

(b) At 10 kHz, can you approximate the ocean as a good conductor, a low-loss medium, or neither?

(c) Assume that the incident wave directly below the airplane is approximately a plane wave with an electric field given in (a). Derive expressions for the E field as a function of depth in the ocean along a vertical line directly below the airplane. You may find the following useful (all units are SI):

Table 7-1: Expressions for α, β, η_c, u_p, and λ for various types of media.

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right]^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f \mu\sigma}$	(Np/m)
$\beta =$	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f \mu\sigma}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu}{\varepsilon'}}\left(1-j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	(Ω)
$u_p =$	ω/β	$1/\sqrt{\mu\varepsilon}$	$1/\sqrt{\mu\varepsilon}$	$\sqrt{4\pi f/\mu\sigma}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

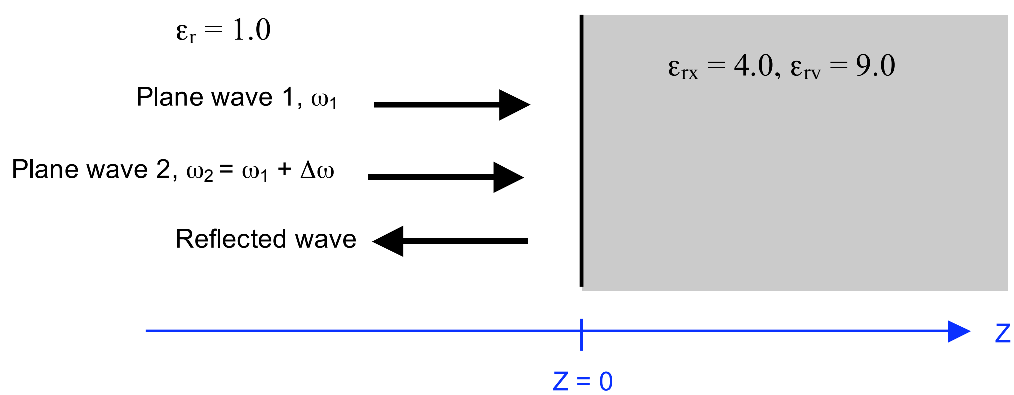
(d) A submarine receiver requires $|E| \geq 10^{-10}$ V/m to detect the radio signal from the airplane. What is the maximum depth of the submarine that still allows it to hear the signal?

7. Two plane waves propagating in the z-direction in free space are normally incident on a birefringent dielectric slab that is infinite in the x-y plane, as shown below. The incident waves are both right hand circularly polarized and have amplitude E_0. Wave 1 has frequency ω_1, while wave 2 has frequency $\omega_2 = \omega_1 + \Delta\omega$, with $\Delta\omega << \omega_1$ (that is, ω_1 and ω_2 are close to each other). The slab has $\varepsilon_{rx} = 4.0$, and $\varepsilon_{ry} = 9.0$.

a) Write down an expression for the total incident electric field, $\vec{E}_i$

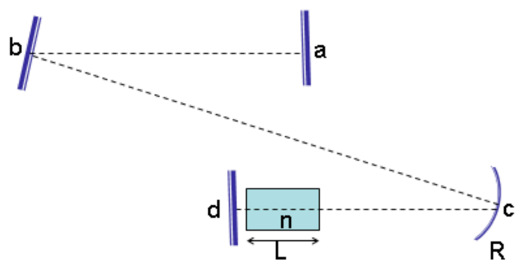
b) Write down an expression for the total reflected electric field, $\vec{E}_r$

c) What is the polarization of the reflected wave?



Lasers – 2010
Answer all 5 questions. All problems count equally. Begin each problem on a new page; put your banner ID at the top of each page. Staple all pages for each problem together, but do not staple your entire exam together.

1. Consider the laser cavity shown below consisting of one concave mirror of radius **R** and 3 flat mirrors. The gain medium having length **L** and index of refraction **n** is inserted very near the flat mirror d. Geometric distances **ab=d$_1$, bc=d$_2$,** and **cd=d$_3$** are known.

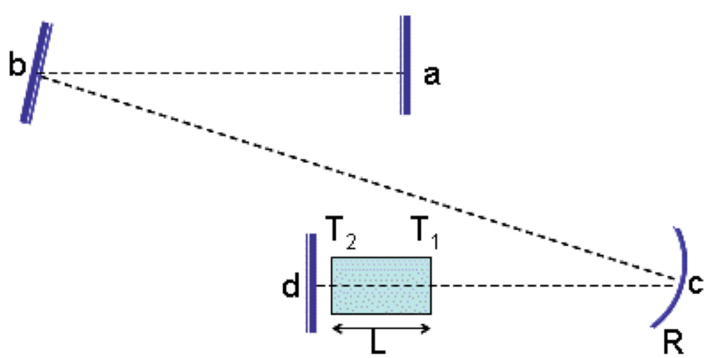


(a) Choose an appropriate starting point and set up the matrix product for evaluating the roundtrip ABCD matrix for this cavity. (Do not multiply the matrices)

(b) Identify the number and the location(s) of the beam waist(s) in this cavity. Explain.

(c) Assuming the ABCD matrix is known from part (a), <u>derive</u> the stability condition for this cavity (only in terms of A, B, C, and D values)?

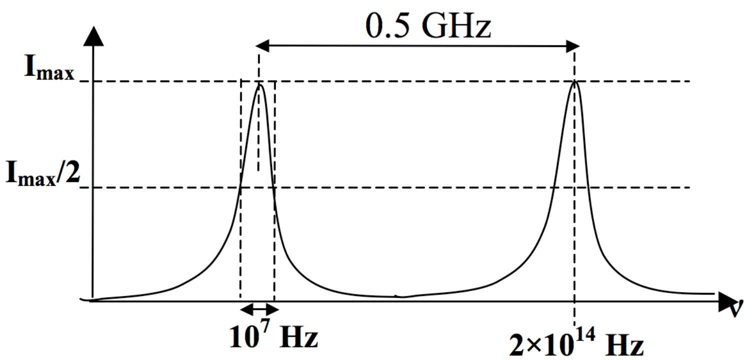
2. Consider the solid-state laser shown below. The mirror reflectivities **R_a , R_b, R_c, R_d ,** surface transmissions of the gain medium **T_1=T_2** and its length **L** are known. Assume beam area **A** inside the gain region. The gain medium is homogeneously broadened with a known saturation intensity **I_s**.

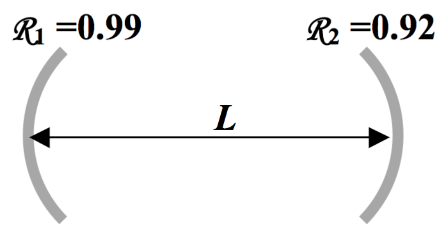


(a) What is the threshold gain coefficient (γ_{th}) ?

(b) Give the cw power (coupled out through mirror d) in terms of the given parameters when pumped **M** times above the threshold (use high-Q cavity approximation).

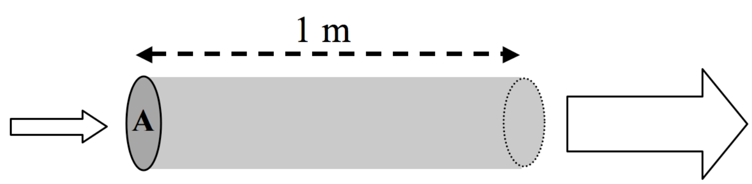
3. The following diagram shows the output spectrum of a laser cavity that is obtained by scanning the wavelength of the input power in the absence of pump power (the peaks are associated with TEM $_{0,0,q}$ and TEM $_{0,0,q+1}$). The space between two mirrors is filled with a dye liquid, which has a refractive index of 1.3.





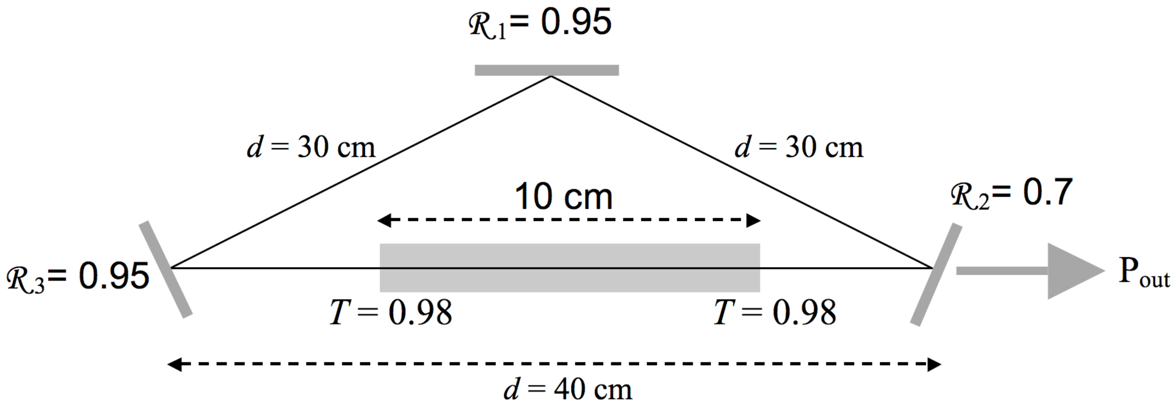
(a) Calculate the cavity length (L), quality factor and the photon lifetime for this cavity.
(b) Calculate the loss factor of the dye liquid (cm^{-1}). Note that $\mathcal{R}_1$ and $\mathcal{R}_2$ are the reflectivity of the mirrors.
(c) If we pump the active medium such that a linear gain of .0023/cm is obtained, how much the quality factor changes?

4. A homogeneously broadened optical amplifier with cross-sectional area of A= 0.5 cm^2 and saturation intensity of 20 W/cm^2 has small signal gain of 10 dB.



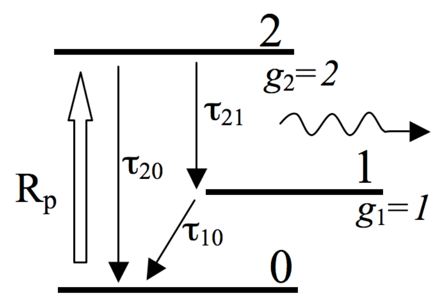
(a) Calculate the small signal gain coefficient (γ_0).
(b) Estimate the maximum power that can be extracted from this amplifier
(c) If the input power is such that the gain is suppressed by a factor of 2 (from its small signal value), compute the output power.

5. Consider the following ring laser:



Properties of the active medium

A_{21}= 5×10^6 s^{-1} (the decay from 2 to 1 is only radiative)
τ_{20} =1 μs
τ_{10} =1 ns
λ_{20} = 0.2 μm
λ_{21} = 1 μm
Refractive index = 3
Homogeneously broadened (Lorentzian line shape)



(a) Calculate the quantum efficiency and stimulated emission cross section (at λ_{21})
(b) Write the rate equations for the active medium in the presence of stimulated emission.
(b) Assuming $N_1 \approx 0$, calculate the threshold excitation rate (R_{th}).
(c) If the beam area (A) is 0.1 mm^2, estimate the CW output power at R_p=4×R_{th}.