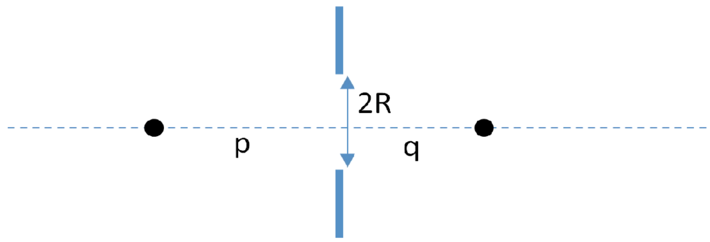
2011 Optical Science & Engineering PhD Qualifying Examination
Optical Sciences Track: Advanced Optics
Time allowed: 90 minutes

Answer all four questions. **All questions count equally.**

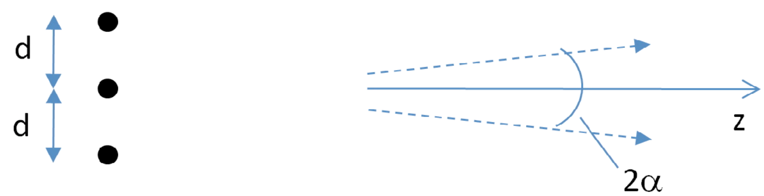
1. A monochromatic point source (wavelength λ) is located a distance p in front of a circular aperture. A point detector is a distance q behind the aperture.
(a) Derive the radius R_n of the n-th Fresnel zone in the aperture plane.
(b) If the aperture has a radius $R = 1$ cm, centered on-axis, how many (half-period) zones does it contain? Use $\lambda = 500$ nm, $p = 20$ cm and $q = 30$ cm.



(c) Every other Fresnel zone is blocked within the aperture. The zone $n = 1$ is open and all zones with $n > 9$ are blocked. Estimate the irradiance at the detector relative to the cases where there is (i) no aperture ($R = \infty$) and (ii) open aperture with $R = R_{10}$.

2. Three small antennas broadcast in phase at a wavelength of $\lambda = 1$ km. The antennas are separated by a distance $d = 2/3$ km. Each point antenna radiates uniformly in all horizontal directions.
(a) What is the angular half width α of the beam centered about the z axis in terms of d and λ in the far field ?
(b) Find a numerical value for α.

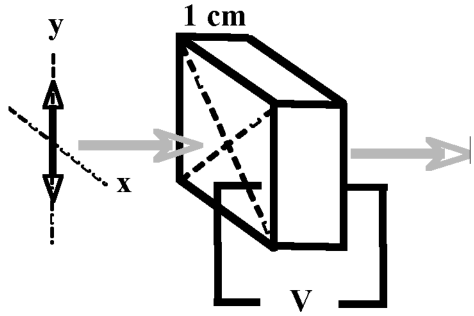
Hint: The broadcast "beam" is limited by interference minima. The half width is defined from the beam center to the first minimum.



(c) Now the antenna to the left (right) broadcasts with a phase shift of $+\pi/9$ ($-\pi/9$) relative to the center emitter. Characterize quantitatively the change in the emission pattern compared to (a) and (b).

3(a) A linearly polarized (along y-axis) beam with $\lambda = 400$ nm enters the optical system shown below. What Voltage should be applied on the Pockels cell so that the transmitted beam is linearly polarized along the x-axis?
(The induced birefringence is along the dotted axes, at 45° to the incident polarization.
The linear electro-optic coefficient of KDP is 11 pm/V and its refractive index is 1.51.)
(b) What would the voltage be if the crystal were 2 cm thick, rather than 1 cm thick?

KDP Pockels cell



4. A partially coherent point source is illuminating a double-slit. (a) Show that the irradiance at an arbitrary point on the screen can be written as:

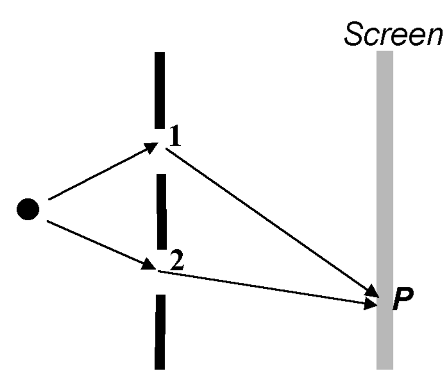
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}[\gamma(\tau)] \qquad \text{where} \qquad \gamma(\tau) = \frac{\varepsilon_0 c}{2}\frac{\langle E(t)E^*(t-\tau)\rangle}{\sqrt{I_1 I_2}}$$

τ is the delay time associated with the difference between the optical path lengths from each hole to the observation point (P), I_1 and I_2 are the irradiances at point P when only one of the slits is open. $E(t)$ is the optical field generated by the point source at P when only one slit is open; assume equal irradiance at P from each hole.

(b) Assuming $\gamma(\tau) = (1-\tau/\tau_c)\, e^{i\omega\tau}$, $\tau<\tau_c$; 0 for $\tau>\tau_c$ and equal irradiance from both holes show that the visibility of the fringes in the mth order is given by:

$$V = 1 - (m\frac{\Delta\lambda}{\lambda})$$

where $\Delta\lambda$ is the line width of the point source.



2011 OSE Ph.D. Qualifying Examination
Electrodynamics

Time Allowed: 90 minutes

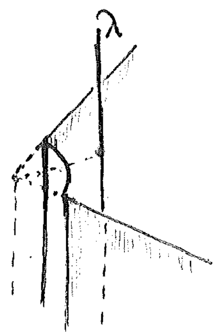
Instructions: Do any THREE problems below. Each problem carries equal weight.

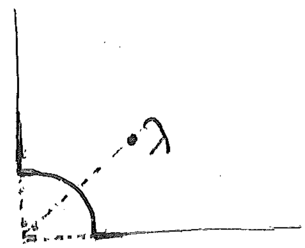
1. Two electromagnetic waves, both at frequency ω and both propagating in air along the z axis, have identical fields at $z = 0$ and $t = 0$, but one is linearly polarized along the x axis, while the other is RCP (right circularly polarized).

   (a) Express the electric and magnetic fields of the superposition of these two waves at any z and t in complex form and also as a real, physical field (i.e., in terms of sin and cos).

   (b) How is the total, time-averaged energy density $\langle w \rangle$ of the superposition of these two waves related to that of the linearly polarized wave alone (find the ratio)?

2. Two conducting planes would intersect normally, were it not for a conducting quarter cylinder of infinite length and radius a that occupies part of the space between the two planes, as shown below. A line charge of density λ per unit length, oriented parallel to the conducting planes, is placed in front of the cylinder a distance d $(d > a)$ from the axis of the cylinder. The planes along with the quarter cylinder are maintained at zero potential.



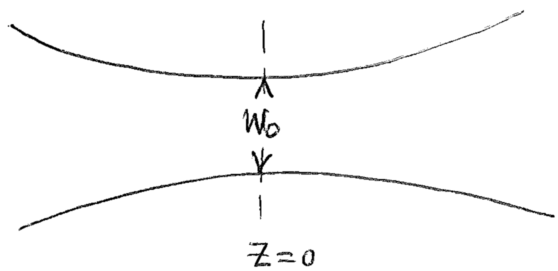


Top View

(a) Show that the electrostatic problem may be solved completely by means of image charges. Where are the image line charges located? How many are needed?

(b) Derive an expression for the electric field everywhere in the quarter space in which the given line charge is located.

3. An electromagnetic beam of frequency ω is focused to a waist of characteristic width w_0 (as measured at its narrowest point) that is located at $z = 0$. The center of the beam propagates along the z axis, and the transverse beam shape is Gaussian at its waist, $\vec{E}(\vec{\rho}, z = 0) = \vec{E}_0 \exp[-\rho^2/(2w_0^2)]$, where $\vec{\rho}$ is the two-dimensional transverse position vector. The beam is linearly polarized at large distances, $|z| \to \infty$.



(a) Show by means of an appropriate Maxwell equation that the field at the waist cannot be purely transverse.

(b) Express the electric field at the beam waist as a Fourier integral, and then propagate it a distance L beyond its waist along the z axis to derive an expression for the field after it has propagated a distance L past the waist location. You may ignore the vector nature of the electromagnetic field and use the Fresnel diffraction approximation for this part of your calculation. (*Hint:* The integral identity,

$$\int_{-\infty}^{\infty} \exp(-au^2)du = (\pi/a)^{1/2},$$

may be useful here.)

4. Consider a rectangular waveguide, infinitely long in the x-directions, with a width (y-direction) 2 cm and a height (z-direction) 1 cm. The walls are perfect conductors. Show all of your work when answering the questions below.

   (a) What are the boundary conditions on the components of the electric and magnetic fields at the walls?

   (b) Write the wave equations which describe the electric and magnetic field components of the lowest mode, (Hint: The lowest mode has the electric field in the z-direction only. Write down equations for E_z, H_x, H_y, and H_z.)

   (c) For the lowest propagating mode, write the general expression for the electric field solution.

   (d) For the lowest propagating mode, find the phase velocity and group velocity.

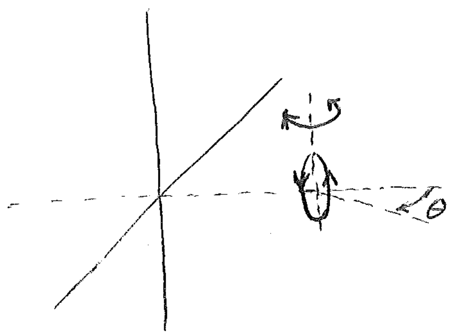
5. A harmonic plane wave of frequency f is incident normally on an interface between two dielectric media with indices of refraction n_1 and n_2, where $n_2 > n_1$. A fraction p of the **energy** is reflected and forms a standing wave when combined with the incoming wave. Recall that on reflection, the phase of the electric field changes by π for $n_2 > n_1$.

   (a) Find an expression for the total electric field as a function of the distance d from the interface. Also determine the positions of the maxima and minima of the total electric field E and calculate the time average $\langle |E|^2 \rangle$ in medium 1.

   (b) From the behavior of the electric field, determine the phase change on reflection of the magnetic field. Find also an expression for the total magnetic field H as a function of distance d from the interface and calculate the time average $\langle |H|^2 \rangle$ in medium 1.

   (c) When O. Wiener did such an experiment, he found in 1890 a band of minimum darkening of a photographic plate for $d = 0$ (at the material interface). Was the darkening caused by the electric or magnetic field and why?

6. A small current loop of radius a carrying a steady current I is a distance d away from a conducting plane. The loop is set into small oscillation at frequency ω about one of its diameters so the angle θ between the loop normal and the plane normal has the time dependent form $\theta = \theta_0 \sin \omega t$. In its equlibrium orientation the loop is parallel to the conducting plane. Assume that $a << 2\pi c/\omega$ and the amplitude of oscillation is small, $\theta_0 << 1$, so the approximations $\sin\theta_0 \approx \theta_0$, $\cos\theta_0 \approx 1$ are well justified.

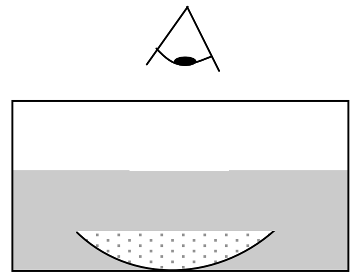


(a) Show that the effect of the conducting plane on the emitted radiation fields can be simulated by an oscillating image current loop. Where is it located and what is the sense of the image current?

(b) Taking d to be larger than the wavelength of emission, determine directions along which no radiation is emitted by the system. (*Hint:* Note that the distances from the two loops to an observation point even in the far field are not the same. Such differences matter in the distance-dependent phase factors, but not in the denominators, of the field expressions.)

2011 Optical Science & Engineering PhD Qualifying Examination
General Optics Knowledge   Time allowed: 180 minutes

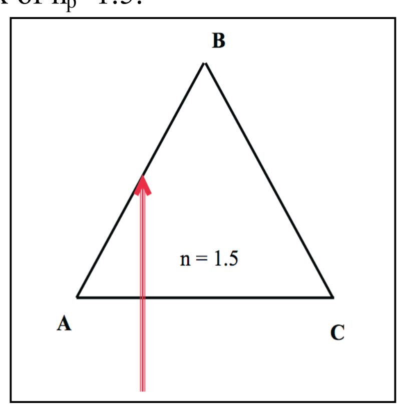
1-A plano-convex lens with a refractive index of 1.5 and power of 0.1 diopter (in air) is placed, convex surface down at bottom of a glass container (that is optically flat) filled with water (n = 1.33). Using a microscope and a sodium lamp (λ = 590 nm), we observe interference fringes from top. Determine the radius of the first dark ring.



2-There is an air bubble under water. A light beam (with a cross sectional area smaller than the bubble) shines through it. After passing through the bubble the light beam converges, diverges or it will be unaffected? Why?

3. The output of a Michelson interferometer is fed to a photodetector.
(a) The input is a (monochromatic) HeNe laser beam of wavelength λ. If one mirror translates at a speed of v, what is the modulation frequency f of the photocurrent ?
(b) Find a numerical value for the modulation frequency f for a wavelength of 633 nm and a translation speed of 5 mm/s.
(c) The maximum mirror translation is 10 cm. What is the minimum resolvable optical frequency $\Delta\nu$ at 500 nm? [Note, $\Delta\nu$ is an estimate of the smallest frequency separation of two monochromatic, mutually incoherent sources near 500 nm that the spectrometer is able to resolve].

4. A narrow beam of light is incident normally on the base of an equilateral prism, as shown. The prism is embedded in a medium whose index can be varied (for example, by changing pressure) from n=1 to n=5. The prism itself has an index of n_p=1.5.



a. What is the angle of the beam exiting the prism through the face AB as a function of the external index n?

b. What is the angle of the beam exiting the prism through the face BC as a function of the external index n?

c. Assume the light is polarized in the plane of the figure, sketch qualitatively the intensity of the beams transmitted through faces AB and BC as a function of the external index n. (Ignore multiple internal reflections.) Be sure to indicate any special points on your graph (for example, where transmission is either 100% or 0%.)

d. Give an equation for n for any special points on your graph.

5. What is the physical meaning of “phase velocity” and “group velocity”?
Derive an expression for group velocity in a dispersive medium. If you like, you may consider the “group” to be two monochromatic plane waves of nearly the same frequency, with the index n a function of frequency $n(\omega)$.

6a. A single, very short pulse plane wave impinges on an infinite slab of glass of thickness t and index n, whose parallel surfaces are partially mirrored to give a transmission of 0.1%. If the pulse impinges normally, what is the period between successive transmitted pulses?

b. Now consider that the plane wave impinges at a small angle theta. (Assume the surface transmission is not changed, and no dispersion.) Now, what is the period between successive transmitted pulses?

7a. A student is given a concavo-convex lens in the lab. In order to identify the surfaces, she looks at the “ghost” reflection of her face from surface A, and then turns the lens around and examines the ghost reflection from surface B. Surface A gives an image smaller than the original and surface B gives a bigger image. Can you make a conclusion about which surface is concave and which is convex?

b. A candle and a screen are separated by 40 cm. When the lens is placed between them, there are two positions of the lens for which a sharp image is obtained on the screen. The lens positions which give sharp images are separated by 20 cm. What is the focal length of the lens?

c. If the bottom half of the lens is covered, how will the image be changed (qualitatively)?

8a.What waveplate would you use to change the polarization of a beam from horizontal to vertical? How would you orient the waveplate?

b. A birefringent material has indices 1.500 and 1.502. What is the thinnest section of this material that can be used as a quarter wave plate for $\lambda = 500$ nm? Is that practical? How can we make a mechanically sound quarter wave plate?

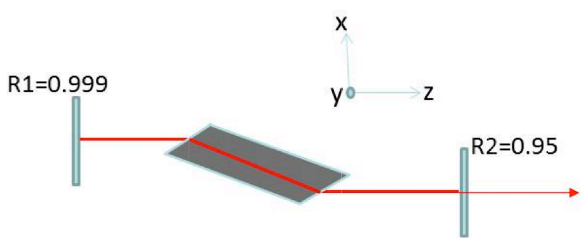
c. What intensity fraction of initially unpolarized light is transmitted by a quarter wave plate?

2011 Optical Science & Engineering PhD Qualifying Examination
Optical Sciences Track: Lasers
Time allowed: 90 minutes

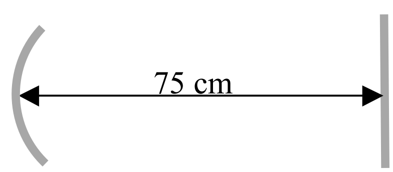
Please answer all four questions. **All questions count equally.**

1. Consider a Yb^{+3} fiber amplifier at λ=1030 nm with a diameter of D = 100 μm and length L_g = 1 m. The concentration of Yb is $N_0 = 1.5 \times 10^{20}$ cm^{-3} and upper state lifetime τ_2 = 3 ms. What is the maximum power that can be extracted from this laser (i.e. the maximum $P_{out} - P_{in}$)? Explain your answer.

2. The solid state laser (shown below) is pumped at 10 × above threshold. If the saturation power of the gain medium inside this cavity is known (P_s = 5W), estimate the output CW power. What is the polarization of the output (x,y, or z, or a combination)?



3. A CO_2 laser operating at λ = 10.6 μm uses a linear cavity made of two mirrors with radii of $R_1 = \infty$, R_2= 5 m (concave), as shown.



Find the spot size on the spherical mirror, in microns (assume $n_{CO2} \sim 1$).

4. The threshold pump power (P_{th}) for the laser in question 3 is measured using two different mirrors as the output couplers with reflectivities R_1 = 90% and R_1' = 95% (they both have the same curvature). The other cavity mirror has reflectivity of R_2 = 100% at the laser wavelength. The measured threshold pump powers are P_{th}= 1W and P_{th}' = 600 mW respectively. Provide an estimate of the roundtrip internal cavity loss.