OSE Qualifying Examinations August 2012 – Optical Science Track – Advanced Optics

Answer all 4 questions; 90 minutes. Be sure to skip parts that you may find difficult so that you can make attempts on all questions before time is called.

Begin each question on a new sheet of paper. Put your Banner ID on each page. Staple all pages of each individual problem together.

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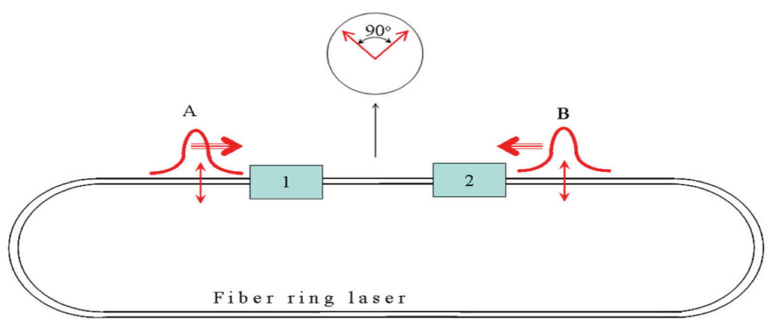
1. Two pulses are on a colliding course, circulating in opposite directions in a ring laser cavity (for example, a fiber ring laser.) The two pulses have identical linear (vertical) polarization. Design a device, consisting of only two elements as sketched in the figure below, such that:

• when the pulses meet, they are cross polarized (with linear polarization)
• after passing through both elements, both pulses are returned to vertical polarization

The devices may not include acousto-optic or electro-optic modulators that are timed to affect the pulses differently.

a) Is there a basic principle of optics that limits the choice of the optical elements?
b) If such elements can (or do) exist, describe how they work.
c) Give one valid reason why one would desire to have crossed polarization at the intersection of the two pulses?

[Note: the pulses would of course collide again at the bottom of the ring! You'd need another pair of devices if you want to avoid "collision" there.]



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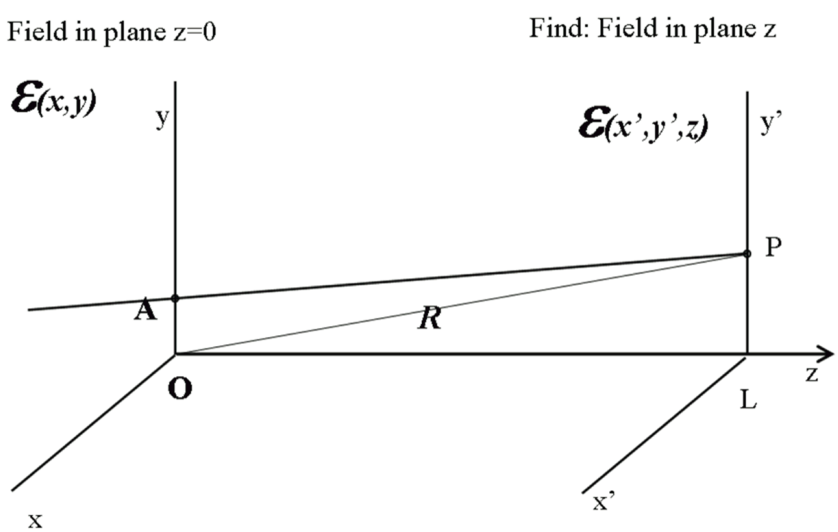
2. An extragalactic radio point source is emitting at a wavelength of 40 cm. The moon passes in front of it, and then it appears again. As it reappears, there are oscillations in its apparent intensity.
a) What are these oscillations caused by?
b) How far does the moon move between the first and second maxima of intensity? You may treat the moon as opaque to radio waves, and assume the moon's edge to be straight. Take the distance to the moon to be 400,000 km. Hint: recall that the Cornu spiral represents the sum of infinitesimal phasors.

3.

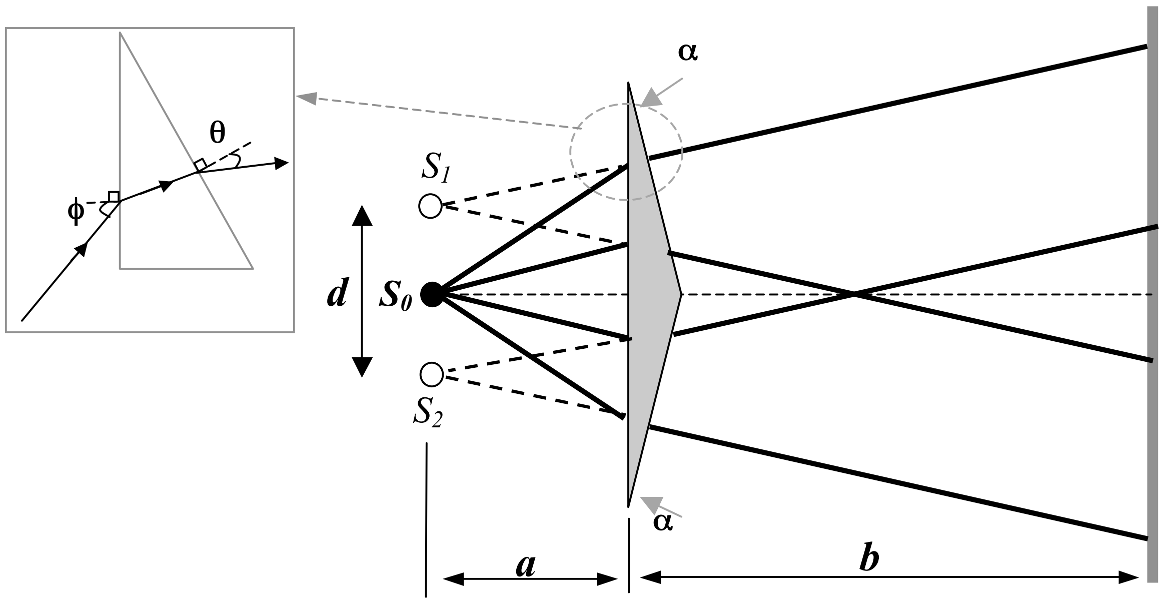
Consider a plane wave with a Gaussian intensity distribution of waist w_0, in a reference plane at $z = 0$. Write the general Huygens integral for the field at a point (x, y) on a surface S at a distance z from this aperture. Express the integral as a product of two functions of x and y.

Find the radial distribution of the electrical field amplitude $\mathcal{E}(x, z)$, using the Fraunhofer approximation.

Find the radial distribution of the electrical field amplitude $\mathcal{E}(x, z)$, using the Fresnel approximation.



4. As shown in the schematic diagram below, a Fresnel biprism acts as a wavefront splitting interferometer and enables a single line source (S_0) to act as two virtual coherent line sources (S_1 and S_2).



a) Assuming the refractive index of the prism is n and α is very small (< 2°) show that $d = 2a(n-1)\,\alpha$

b) If $a = 25$ cm , $b = 75$ cm, $n = 1.7$ and $\alpha = 0.9°$ find the separation between the fringes generated by monochromatic source with a wavelength of 600 nm.

OSE Qualifying Examinations August 2012- General Optics – All Tracks 180 minutes.

Answer all questions. Be sure to skip parts that you may find difficult so that you can make attempts on all questions before time is called. All questions count equally, except Q11, which counts double.

Begin each question on a new sheet of paper. Put your Banner ID on each page. Staple all pages of each individual problem together.

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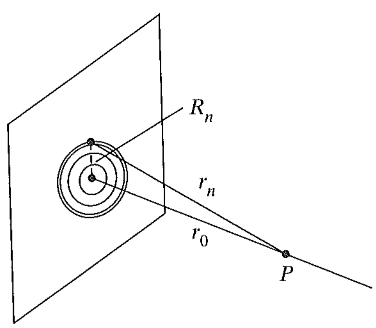
1. A student seeks to make a better Fresnel-type zone plate to focus light at a point. Rather than use an approximate expression for the the radius of the nth zone, the student computes the exact radius.

a) What is the exact radius of the first zone, R_1?
b) Is this radius larger or smaller than that of the standard Fresnel 1st zone?
What approximation is used to obtain the standard Fresnel zone radii?



2.a) In one sentence, describe what is meant by "blazing" a diffraction grating.

b) A reflection grating has 500 grooves/mm when viewed at normal incidence. At what angle(s) is 200 nm light reflected in 2nd order?

c) What blaze angle should be used to obtain maximum intensity in the 2nd order reflection for 200 nm light, with normal incidence?

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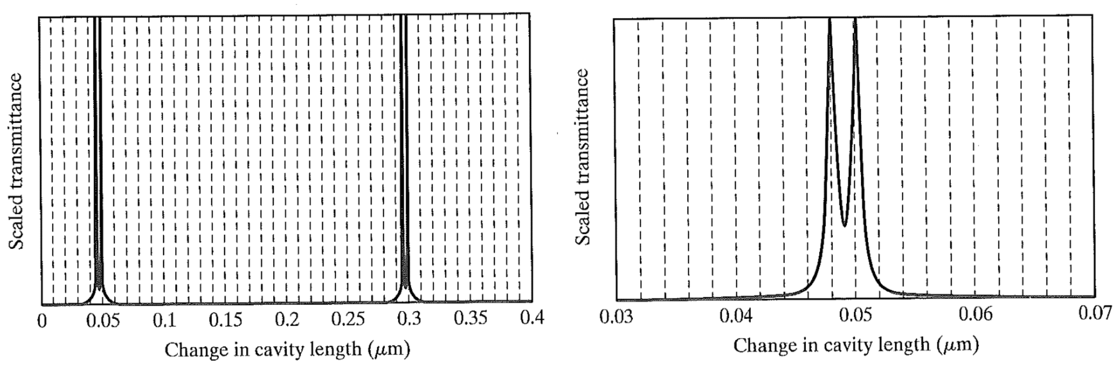
3. Suppose the red line from a low pressure cadmium lamp at 648.3 nm gives visible fringes in a (single pass) Michelson interferometer while the movable arm is moved through a distance of 10 cm.
a) Explain in words why the fringes disappear if the arm is moved farther.
b) What is the width of the line, in nm?
c) What is the coherence time of this line?

4. A light source consisting of two components with closely spaced wavelengths is normally incident on a scanning Fabry-Perot interferometer with nominal length d=2 cm. The scaled transmittance is shown in the Figure, blown up on the right.

a) What is the nominal wavelength of the source?

b) Estimate the difference in wavelength of the two components, assuming they have the same mode number.

c) Estimate the different in wavelength of the two components, assuming the right peak has a mode number one larger than that of the left peak.



5. A refracting telescope is to be built from a 5" diameter lens of focal length +40 cm, and a ½" diameter lens of focal length +2 cm.

a) How far apart should the lenses be placed?
b) What magnification will be achieved?
c) What is the "aperture stop" for this optical system?
d) What is the "exit pupil"?

The dark-adapted eye has a pupil with a diameter a little larger than ¼"; in daylight, the diameter is a bit less than 1/6" of an inch.

e) Is this telescope better designed for astronomical (nighttime) or military (daytime) use?
f) In a sentence, mention an advantage or two of reflecting telescopes over refractors.

6. A film is made from soapy water across a circular loop of wire. As the film drains, colors are visible in reflection, when the film is illuminated by white light, but soon the top part becomes invisible (i.e. dark, in reflection.)

a) Why is the top part dark?
b) Looking at the highest part of the film that is not dark, what color is it?
c) The thickest part of the film shows no colors. Why?
d) You notice that green bands appear 1 mm apart on the film. What is the approximate wedge angle of the draining film? (Treat soapy water as water, for optical purposes.)

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7. a) Make a rough estimate of the distance from which car headlights can be resolved by the human eye.

b) Suppose one wants to distinguish a car from a motorcycle at 10 times this distance. Could one use a Young's double slit experiment to do so (with a suitably sensitive detector for the interference pattern?) If so, how far apart do the slits need to be?

c) Your eyes are considerably farther apart than the size of your pupils. However, your retina only records light intensity, not phase. You are called in to consult on building a bionic neuroimplant that would allow a human to estimate the spatial extent of a very distant source by combining optical signals from both eyes. Without phase information, can this work?

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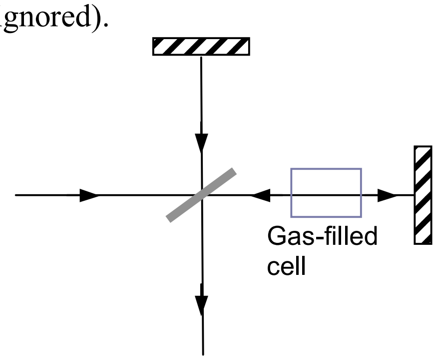
8. A gas-filled cell of length 5 cm is inserted in one arm of a Michelson interferometer, as shown in the figure below. The interferometer is in vacuum and is illuminated by light of wavelength 500 nm. As the gas is evacuated from the cell, 40 fringes cross a point in the field of view, Estimate the refractive index of this gas (the thickness of the splitting mirror can be ignored).



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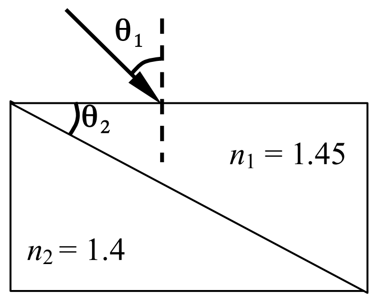
9.A linearly polarized optical beam is incident on a dielectric block (shown below) immersed in water (n_{water}=1.33). Determine the polarization of the input beam (TE or TM, or s or p), θ_1 and θ_2 such that the first reflection is zero and the second reflection is 100%. Sketch the polarization on a drawing.



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10. Unpolarized light is incident on two ideal polarizers in series. The polarizers are oriented so that no light emerges through the second polarizer. A third polarizer is now inserted between the first two and its orientation direction is continuously rotated through 180°. What is the maximum fraction of the incident power transmitted through all three polarizers, and at what orientation is this maximum achieved?

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11. (Counts double) Answer any 5 of the following questions (i.e. you may skip 3 of them.)

a) What is the physical meaning of phase velocity and group velocity?
b) Describe two methods for mode locking a laser
c) Describe two methods for Q-switching a laser
d) Explain the operation of an edge-emitting diode laser
e) Why might your grandmother squint when threading a needle?
f) What waveplate could you use to change polarization from horizontal to vertical without loss? How should it be oriented?
g) Explain why Brewster windows are sometimes used in lasers.
h) What should be orientation of the transmission axis on polarized sunglasses? Why?

OSE Qualifying Examinations August 2012
Optical Science Track – Lasers

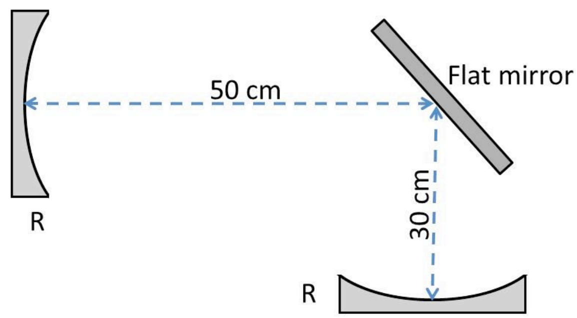
Answer all 4 questions; 90 minutes. Be sure to skip parts that you may find difficult so that you can make attempts on all questions before time is called.

Begin each question on a new sheet of paper. Put your Banner ID on each page. Staple all pages of each individual problem together.

1. Consider the linear (standing-wave) cavity shown below. The two concave end mirrors have the same radius of curvature R.

(a) Derive the stability condition (in terms of R) for this cavity.

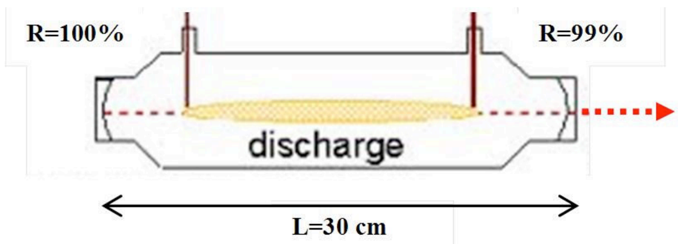
(b) Obtain the position and magnitude of the minimum beam waist (w_0) in terms of R. Assume λ=1 μm.



2. Consider a homogenously broadened CW gas laser (shown below) pumped 9x above threshold.

(a) Estimate the output power given the saturation power is 10 mW.

(b) Estimate the peak power and pulse duration if this laser is mode-locked with N=1000 longitudinal modes.



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3. Consider the case of a transition that is broadened such that the magnitude of the gain peak $g(\nu_0)$ is ~ 5 times larger than that of the total intracavity loss. Assume that only TEM_{00} modes are allowed to oscillate, and that the longitudinal mode separation is only 0.3 times the FWHM (full-width half maximum) of the transition linewidth, and that one of these longitudinal modes coincides in frequency with the gain peak.

a) ***Homogeneously broadened*** line: Plot the gain and loss curves for the unsaturated case, and then add the gain curve for the saturated case to this plot, and discuss the lineshapes and saturation behavior in no more than 2 short sentences.

b) ***Inhomogeneously broadened*** line: Plot the gain and loss curves for the unsaturated case, and then add the gain curve for the saturated case to this plot, assuming that the homogeneous linewidth for the gain atoms in this system is equal to 10% of the inhomogeneous linewidth. Discuss the lineshapes and saturation behavior in no more than 3 short sentences.

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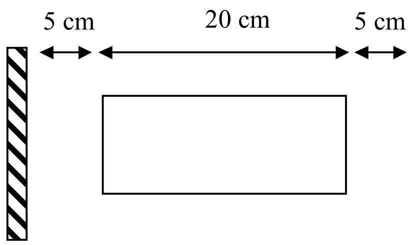
4. The cavity shown below has quality factor (Q) of 3×10^7. The laser utilizes an atomic transition in the active medium that peaks at 0.55 μm with $A_{21} = 4\times10^5$ sec^{-1}. The degeneracy of the upper and lower levels is 5 and 3 respectively. The active material has a refractive index of 1.3 with an inhomogeneous line shape that can be approximated by the graph shown.

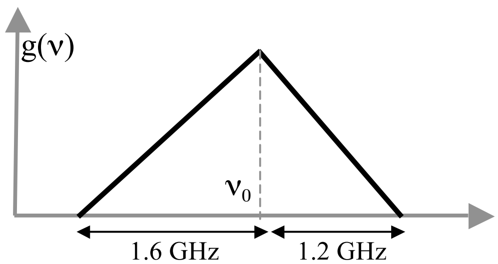
(a) Estimate the photon lifetime in the passive cavity.

(b) Calculate the stimulated emission cross section for this laser.

(c) Assuming the population of the state-1 is 10^{12} cm^{-3}. Calculate the population of state- 2 to reach threshold.

(d) If we pump this laser such that small signal gain coefficient of the active medium is three times larger than its threshold value, how large is the intensity of the circulating optical power inside the cavity compared to the saturation intensity.





Fall 2012
OSE Qualifying Examination
Classical Electrodynamics

August 17, 2012

Instructions: **Solve any 3 problems. All problems carry equal points.**

1. A right-circularly polarized, monochromatic electromagnetic (EM) beam of frequency ω propagating in the z direction in free space has a finite but large (on the wavelength scale) transverse extent, so its electric field (in complex notation) may be expressed as

$$\vec{E}(x,y,z,t) = [\hat{e}_+ E_0(x,y,z) + \hat{z}E_z(x,y,z)]\exp(ikz - i\omega t), \quad \omega = ck$$

with its transverse amplitude E_0 having a non-trivial x, y dependence, $\partial E_0/\partial x$, $\partial E_0/\partial y \neq 0$. Here $\hat{e}_+ = (\hat{x} + i\hat{y})$.

(a) Show by applying $\vec{\nabla} \cdot \vec{E} = 0$ that the longitudinal component of $\vec{E}$ cannot vanish, $E_z \neq 0$, and is given approximately by

$$E_z = \frac{i}{k}\left(\frac{\partial E_0}{\partial x} + i\frac{\partial E_0}{\partial y}\right).$$

Throughout this problem, assume that the first-order z-derivatives of E_0 and E_z obey the inequalities,

$$\left|\frac{\partial E_0}{\partial z}\right| << k|E_0|, \quad \left|\frac{\partial E_z}{\partial z}\right| << k|E_z|,$$

and thus are negligible. This is known as the paraxial approximation.

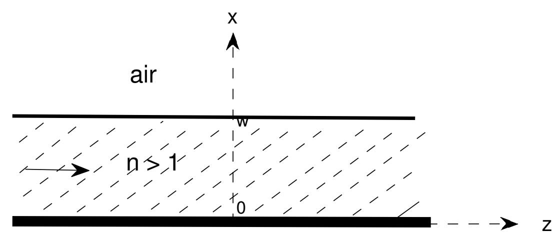
(b) Derive, by using Faraday's law, an expression for the magnetic field, $\vec{B}$, of the EM beam in the paraxial approximation, keeping only terms of order 1 and $1/k$ in the amplitudes. Show that $\vec{B} \approx -i\sqrt{\mu_0\epsilon_0}\vec{E}$ in this paraxial approximation.

(c) Consider the special case (an example of an orbital-angular-momentum (OAM) beam) where

$$E_0 = A(x - iy),$$

where A is a real constant. Write down expressions for $\vec{E}$ and $\vec{B}$ using results of part (a) and (b). Then calculate the time-averaged Poynting vector $\vec{S}$ for this case. Show by expressing $\vec{S}$ in cylindrical coordinates (ρ, ϕ, z) that for fixed distance from the z axis, *i.e.* for fixed ρ, this vector spirals around the z axis.

2. The bottom surface of a dielectric slab waveguide of thickness w and refractive index $n > 1$ is metallized to make it perfectly reflecting, while its top surface is directly in contact with air (refractive index = 1). Take the bottom surface to be in the yz plane.



(a) What are the different types of monochromatic waveguide modes possible in such a guide?

(b) Construct, by solving appropriate wave equations, the spatial distribution of the electric field of a TM mode that is propagating along the z axis inside the guide.
(*Hint:* The z component of the electric field, E_z, can be chosen to have the form $f(x)\exp(ikz - i\omega t)$ that obeys two different scalar Helmholtz equations, one inside the dielectric and a different one in the air. The other components of the electromagnetic field are related to E_z. Specifically, E_x may be expressed in terms of E_z as

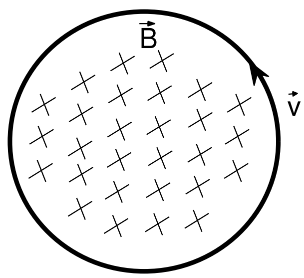
$$E_x = \begin{cases} \dfrac{ik}{\gamma^2}\dfrac{\partial E_z}{\partial x} & \text{for } 0 \le x < w \\ \dfrac{-ik}{\beta^2}\dfrac{\partial E_z}{\partial x} & \text{for } w < x < \infty, \end{cases}$$

where γ and $i\beta$ are the transverse propagation constants of the guided mode inside the guide and in air, respectively, *i.e.,* $\gamma^2 = \omega^2 n^2/c^2 - k^2$, $\beta^2 = k^2 - \omega^2/c^2$.)

(c) Derive the eigenvalue equation for the TM mode by imposing appropriate boundary conditions on the electric and displacement fields at the bottom surface ($E_z = 0$ there) and at the air-dielectric interface (continuity of E_z and of D_x).

(d) By considering the eigenvalue equation graphically, show that the TM modes do not have a cutoff frequency. Explain in physical terms why that must be so in such a "one-sided" guide.

3. A point charge q of mass m moves on a circle of initial radius R under the Lorentz force exerted by a uniform static magnetic field $\vec{B}$ that is orthogonal to the plane of the circle. Neglect all other forces on the charge and take its motion to be non-relativistic.



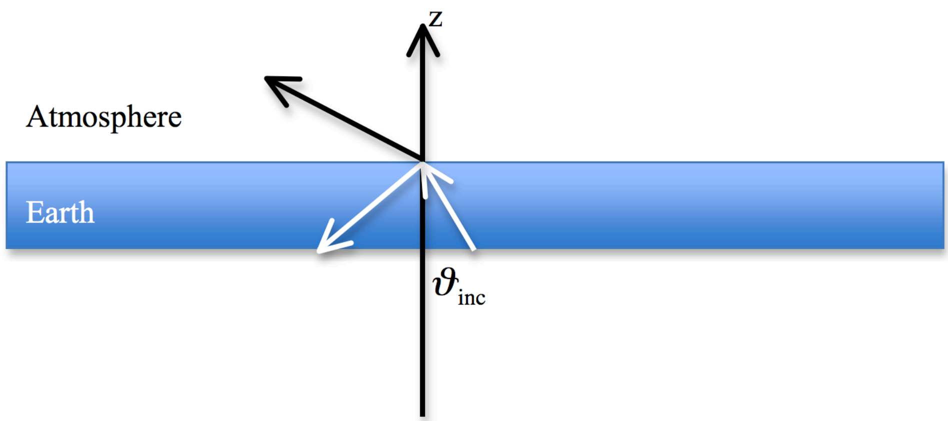
(a) What is the speed of the charge? What is its acceleration a? What is its total mechanical energy?

(b) What is the rate at which the charge loses energy by radiation? (*Hint:* Larmor formula for radiated power: $P = \mu_0 q^2 a^2/(6\pi c)$.)

(c) How long will it take for the orbital radius of the radiating charge to decrease to R/e?

(d) Explain how your answers to parts (a)-(c) will change, calculating explicitly those changes, if the charge had some initial out-of-plane velocity component.

A LHCP (left hand circularly polarized) wave is propagating from the interior of a lossless Earth (medium 1, $\sigma = 0$) with index of refraction $n = 3$ to lossless atmosphere (medium 2). The wave is incident onto the planar Earth-atmosphere interface from the Earth at an angle of ϑ_{inc}=18.4349 degrees and is partially reflected and partially transmitted, as shown in the sketch below. Note that $\mu_1 = \mu_2 = \mu_0$.



a) What is the polarization of the wave that is reflected at the interface back into Earth (circular, elliptical, linear)? [2.5 points]
b) If rotating, what is the parity or sense of rotation of the reflected wave? [2.5 points]
c) What is the polarization of the transmitted wave propagating skyward? [2.5 points]
d) If rotating, what is the parity or sense of rotation of the transmitted wave? [2.5 points]

Formulae that you might find useful:

$$T_{\perp}^{b} = \frac{2\cos\vartheta_{\text{inc}}}{\cos\vartheta_{\text{inc}} + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\sqrt{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)\sin^2\vartheta_{\text{inc}}}}$$

$$T_{\parallel}^{b} = \frac{2\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\cos\vartheta_{\text{inc}}}{\cos\vartheta_{\text{inc}} + \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sqrt{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)\sin^2\vartheta_{\text{inc}}}}$$

$$\vartheta_{\text{Brewster}} = \tan^{-1}\left(\frac{\mathbf{n}_2}{\mathbf{n}_1}\right)$$

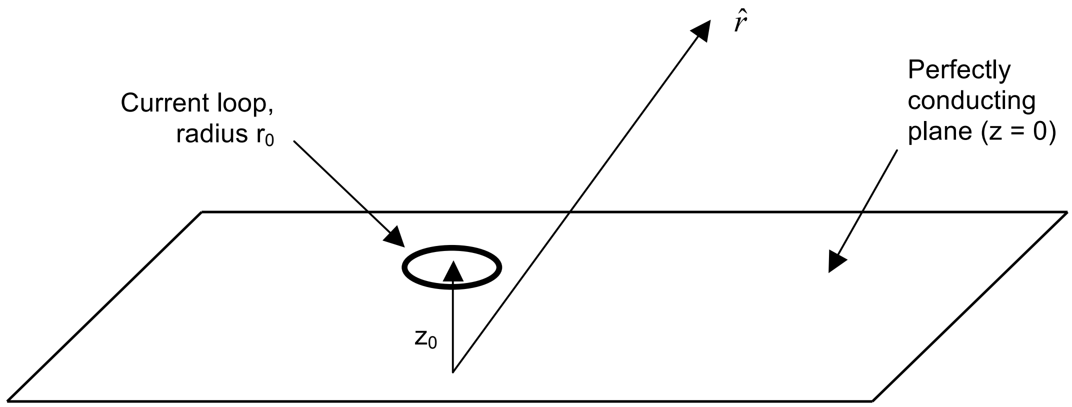
$$\vartheta_{\text{Critical}} = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

The sun irradiates the Earth at a rate of 2.00 cal/cm^2 per minute. This rate is known as the solar constant.

a) What is the magnitude of the Poynting vector (in W/m^2) in sunlight? [2 points]
b) What is the magnitude of the rms electric field (in V/m) in sunlight? [2 points]
c) What is the magnitude of the rms magnetic field (in A/m) in sunlight? [2 points]
d) What is the power output of the sun in sunlight alone (in W)? [2 points]
e) Why do you get sunburned if you are exposed to sunlight for a period of time without sunscreen (and particularly at high altitude such as Albuquerque, NM)? [2 points]

Some useful info: 1 cal=4.186 J; the distance from the sun to Earth is 1.50x10^{11} m.

A small current loop of radius r_0, carrying current $\vec{I} = \hat{\phi} I_0 e^{j\omega t}$ is centered at position x = 0, y = 0, z = z_0 over an infinite, perfectly conducting plane, located at z = 0, as shown below.



a) Find the far-field vector potential $\vec{A}$ at a position $\vec{r}$ above the conducting plane (i.e. z > 0).

b) Find the electric field, $\vec{E}$, at the position $\vec{r}$ in the far field.

c) Now consider the current loop moving upward in the z-direction with constant velocity $\vec{v} = \hat{z} v_{z0}$. Find $\vec{A}$ at a position $\vec{r}$.