Optical Science and Engineering – 2013 Advanced Optics Exam
Answer all questions. Begin each question on a new blank page. Put your banner ID at the top of each page. Please staple all pages for each individual question together.

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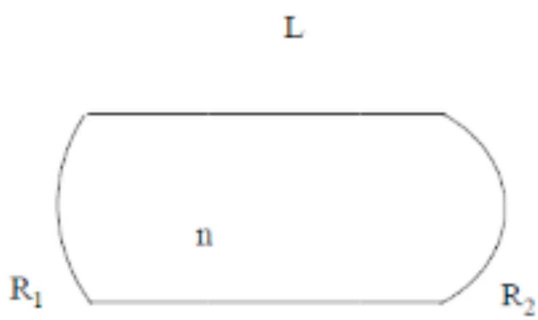
1. Consider the biconvex thick lens shown in the figure below, made from transparent material with index n and thickness L.

a. <u>Derive</u> the transfer ray matrix for this lens.

b. In terms of the radii of curvature, and n, obtain the thickness L for which the lens will act as a telescope for paraxial rays. (In other words, you want parallel entering rays to exit parallel.) Write the ray matrix for this case.

c. If the telescope is used as a beam expander to double the diameter of a collimated beam, what condition must be satisfied by R_1 and R_2 ?

d. A collimated laser beam with diameter d is incident on the beam expander. Sketch the path of a couple of rays through the system, and identify the intermediate focus, if there is one. If not, explain why not.



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2. Two different electromagnetic plane waves have electric fields given by

$$\vec{E}_1(t) = \left(\hat{a}_x + \hat{a}_y\right) E_0 \cos(\omega t)$$
$$\vec{E}_2(t) = \left(\hat{a}_x - \hat{a}_y\right) E_0 \cos((\omega + \Delta\omega) t)$$

at their point of incidence with a zero order quarter wave plate. Both waves are propagating into and normal to the plate. $\Delta\omega << \omega$. The fast axis of the quarter wave plate is along x.

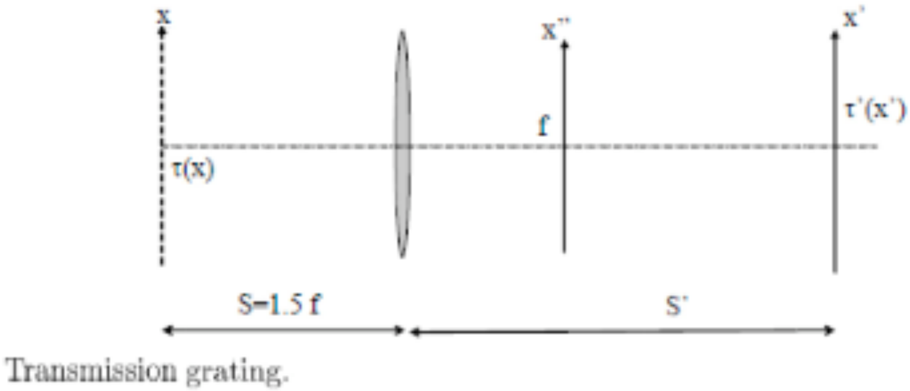
a. Find an expression for the electric field of the total combined wave immediately after passing through the wave plate.

b. Describe the polarization of the output beam (i.e. linear, circular, elliptical, orientation, etc.). Does it change in time? If so, describe how (in words.) If not, explain why not.

3. A transmission grating having a field transmission function

$$\tau(x) = \cos^2(\pi x / \Lambda)$$

is illuminated by a coherent plane wave of wavelength $\lambda << \Lambda$. The grating is imaged through a single thin lens (of effectively ∞ diameter) as shown in the figure.

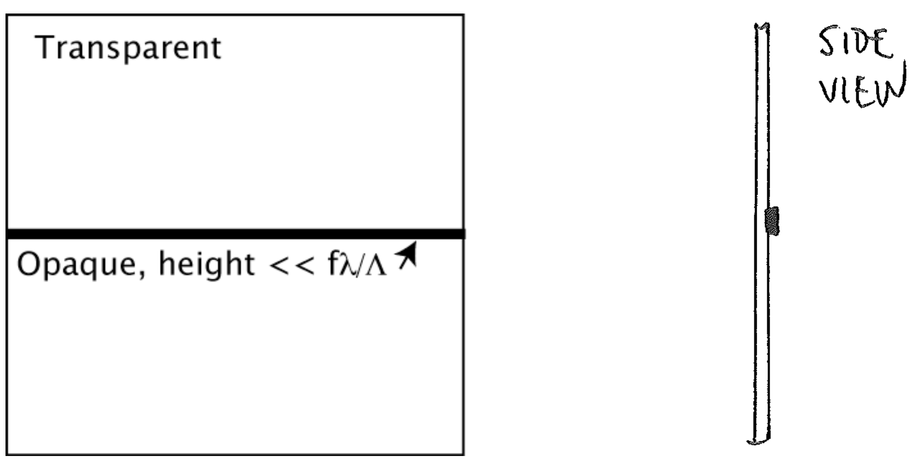


Transmission grating.

a. Derive the field amplitude distribution, as a function of x', in the image plane. How does it compare with the transmission function?

b. Derive the field amplitude distribution in the focal plane, as a function of x''.

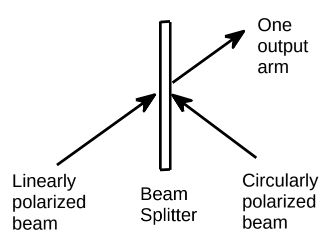
c. Derive the field amplitude distribution in the image plane as a function of x', if the filter below is placed in the focal plane, perpendicular to the optical axis. The filter has a thin opaque center strip.



Fall 2013
OSE Qualifying Examination
Classical Electrodynamics

Instructions: Solve any 3 problems. All problems carry equal points.

1. A linearly polarized plane wave and a right-circularly polarized plane wave, both of identical intensity I_0 and angular frequency ω, are combined at a 50-50 beam splitter at which they are each incident at 45^0 on its two sides. The beam splitter has a highly random jitter, which scrambles the relative phase between the two beams upon combination.

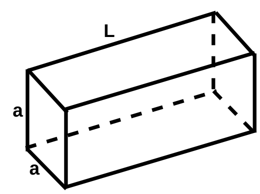


   (a) Express the electric and magnetic fields of the radiation field in one of the two output arms of the beam splitter. Denote the uniformly random relative phase between the two combined beams to be ψ, and the output field under consideration to be propagating along the z axis in free space. (*Hint:* Pay attention to factors of $\sqrt{2}$ when calculating the amplitudes of the two beams in terms of I_0.)

(b) Derive an expression for the time-averaged Poynting vector of the outgoing radiation in terms of I_0 and the usual electromagnetic constants.

(c) Calculate the coherency matrix $\mathbf{J}$ for the outgoing field and its eigenvalues. Then calculate its degree of polarization, $\mathcal{P}$, which is the ratio of the difference and sum of those eigenvalues. (*Hint:* The ij element of the coherency matrix is defined as the phase averaged value of the product $E_i E_j^*$, where $i, j = 1, 2$ denote any two mutually orthogonal linear-polarization directions.)

2. A hollow pipe of length L and square cross-section of side a is terminated by square faces of side a that are orthogonal to its length dimension which is along the z axis. The walls of the so-formed cavity are perfectly conducting, and the medium filling the cavity is lossless and with a real dielectric permittivity, ϵ'.



(a) Calculate the spatial distribution of the longitudinal component of the electric field for the various TM modes of the cavity.

(b) What is the resonant frequency of each such mode?

(c) Derive an expression for the $\vec{H}$ field for the lowest-frequency TM mode inside the cavity.

(d) Show that the time averaged electromagnetic energy inside the cavity in the lowest-frequency TM mode has the value

$$U = \frac{L}{8}\epsilon' a^2 |\psi_0|^2,$$

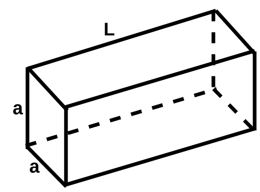
where ψ_0 is the maximum amplitude of the electric field inside the cavity.

Hints: (a) Time-averaged electromagnetic energy density - $(1/4)(\epsilon'|E|^2 + \mu_0|H|^2)$.
(b) The transverse $\vec{E}_T$ field is related to the combined longitudinal-transverse gradient of the longitudinal component of the electric field as

$$\vec{E}_T = \frac{1}{\gamma^2}\frac{\partial}{\partial z}\vec{\nabla}_T E_z,$$

where γ is the so-called transverse propagation constant.

3. Consider the previous problem for the lowest-order TM mode, namely TM_{110}. For this mode, the fields can be shown to have no z dependence, and the electric field is along $\hat{z}$ alone where $\hat{z}$ is parallel to the cavity length.



(a) The electric field in this mode has the form

$$\vec{E} = \hat{z}\psi_0 \sin(\pi x/a)\sin(\pi y/a)\exp(-i\omega t),$$

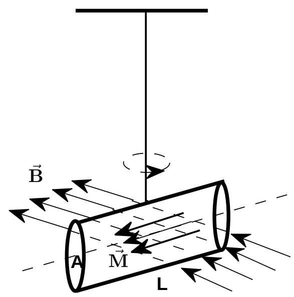
where ω is the resonant frequency of this mode. Write down an expression for ω simply by noting that $\vec{E}$ must obey the wave equation inside the dielectric medium. Assume perfectly conducting walls and vanishing ϵ'' for this calculation.

(b) From the expression for $\vec{E}$ in part (a) and an appropriate Maxwell equation, show that the $\vec{H}$ field has the form

$$\vec{H} = \frac{-i\pi}{a\omega\mu_0}\psi_0[-\hat{y}\cos(\pi x/a)\sin(\pi y/a) + \hat{x}\sin(\pi x/a)\cos(\pi y/a)]\exp(-i\omega t).$$

(c) Allowing now for a finite but large conductivity σ for the cavity walls and a slighly lossy dielectric with permittivity $\epsilon' + i\epsilon''$, with $0 < \epsilon'' << \epsilon'$, derive an expression for the cavity Q factor for this mode. You may use any expressions you may like from the statement of the previous problem without proving them. (*Hint:* The time-averaged power loss per unit area of a highly conducting wall is $|H|^2/(2\sigma\delta)$, where δ is the so-called skin depth. The time-averaged power loss per unit volume of a lossy dielectric is simply $(1/2)\omega\,\mathrm{Im}(\vec{P}\cdot\vec{E}^*)$, where ω is the angular frequency of the electromagnetic field.)

4. A small cylindrical bar magnet of length L and uniform cross-section of area A carries a uniform magnetization $\vec{M}$ along its length. It is placed in a uniform static magnetic field $\vec{B}$ that is orthogonal to $\vec{M}$. Both these directions are parallel to the horizontal xy plane.



(a) What are the force and torque that the magnet experiences?

(b) If the magnet is allowed to come to rest in its stable equilibrium orientation in the xy plane and then suspended at its center of mass by a torsion wire along the vertical direction, then calculate the frequency with which the bar magnet will perform simple harmonic oscillations in the xy plane if turned by a small angle θ_0 from its equilibrium orientation in that plane. Take the moment of inertia of the bar magnet about an axis through its center of mass and along $\hat{z}$ to be I, and the restoring torque of the suspending wire to be $-\gamma\theta$ for an angular deviation of amount θ. (Small-angle approximation: $\cos\theta \approx 1,\ \sin\theta \approx \theta$.)

(c) What is the time rate at which energy is being radiated by the oscillating bar magnet, assuming that its dimensions are small compared to the wavelength of radiation? Estimate the characteristic time over which the oscillating magnet will lose a significant portion of its mechanical energy, assuming no other mechanisms for energy loss except radiation. (*Hint:* The radiated power formula for magnetic-dipole radiation is the same as that for electric-dipole radiation except for the substitution $\vec{p} \to \vec{m}/c$.)

5.(a) Write down expressions for solutions to the Helmholtz equation in cylindrical geometry that represent radially outgoing and radially incoming traveling wave solutions. Assume that the propagation medium is lossless and homogeneous.

(b) The solution to the Helmholtz equation in cylindrical geometry that represents radially outgoing traveling wave solutions reduces to cylindrical wave functions in the asymptotic (large $\beta\rho$) limit. Write down the functional dependence for cylindrical wave functions.

(c) How would your answer to part (a) change if the propagation medium were now a lossy medium?

(d) In the limit of low but finite losses, derive an expression for the characteristic distance over which the cylindrical wave functions decay in the asymptotic limit.

Explicitly define all of the variables you are writing down in your responses (e.g., β is the propagation constant, ρ is the radius in cylindrical coordinates).

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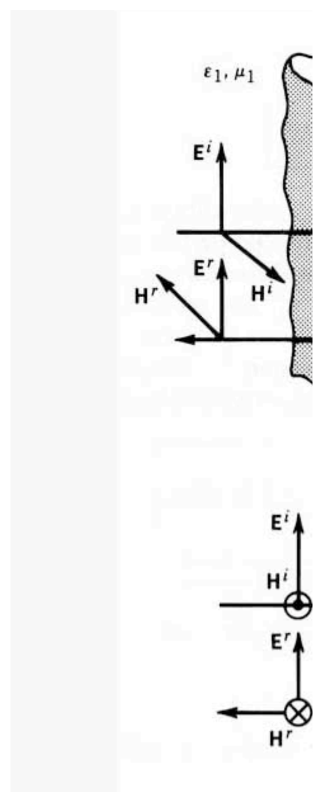
6. Consider the interface between two lossless regions as shown in the figure.
A linearly polarized TEM plane wave is normally incident from region 1 to the interface as shown in the figure.

(a) Derive the Fresnel reflection and transmission coefficients for this situation.

(b) Under what circumstances does the electric field polarization flip for the transmitted wave, and under what circumstances does it not flip? Would your answer change if the incident wave were RCP (right circular polarized) ?

(c) If $\mu_1 = \mu_2 = \mu_0$ write down an expression for the standing wave ratio (SWR) for the case when $\varepsilon_1 > \varepsilon_2$ and for the case when $\varepsilon_1 < \varepsilon_2$.
SWR = ratio of intensity at an antinode to that at a node.



7. It is well known that magnetic fields penetrate conductive media by diffusion.
(a) Use Faraday's and Ampere's Laws to derive the diffusion equation for magnetic field,

$$\frac{\partial}{\partial t}\vec{B} = \frac{\eta}{\mu_0}\nabla^2\vec{B},$$

where η is the resistivity of the medium. <u>Hint</u>: neglect displacement current.

(b) Now consider a magnetic field diffusing into a conductor at its interface with free space. Consider a one dimensional case. Assume that $B(x,t) = B_0 e^{-x/L} f(t)$, where B_0 is an initial field, and f(t) is a function of time. Find the solution to the diffusion equation in this case.

(c) It is also well known that magnetic fields cannot penetrate perfect conductors. If a time-magnetic field is incident on a perfect conductor boundary (as in part b), what happens physically to exclude it (the magnetic field) from the conductor?

(d) Include now the displacement current as well. How does your answer for part a) change? Consider now a monochromatic solution of the form $B(x,t) = B_0 f(x) e^{-i\omega t}$ for the more general equation obeyed by B and evaluate f(x) for a forward propagating wave.

OSE 2013 Examinations – General Optics 3 hours.

Answer all questions. Begin each question on a new sheet of paper. Put your banner ID at the top of each sheet. Most problems should only require one sheet of paper, but if you use additional sheets, staple all sheets for each individual problem together.

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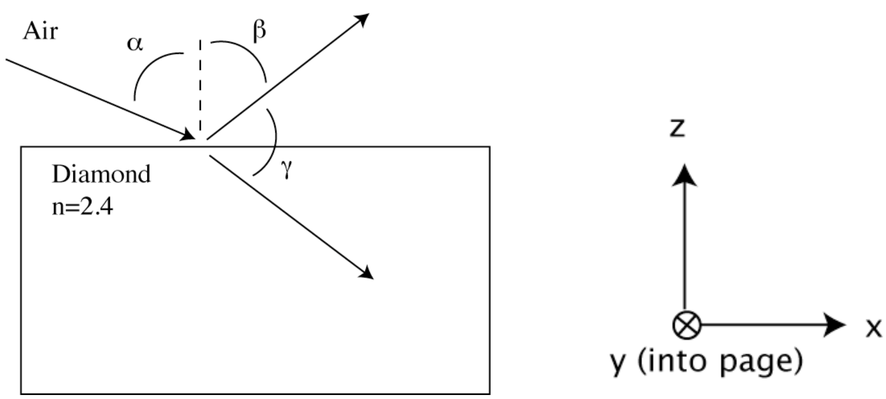
1. Unpolarized light in air is incident on a flat surface of a diamond. The reflected light is completely polarized.

a. To the nearest degree, what are the angles α, β, γ in the figure?

b. What is the nature and orientation of the polarization (electric field orientation) for the reflected light? Express in terms of the coordinate system given below.



2.A circular aperture of diameter 20 microns is illuminated normally with a plane wave of wavelength 500 nm. Make a rough sketch of the intensity along the axis past the aperture, from close to the aperture out to a large distance (i.e. out to where far-field diffraction is valid.) Are (is) there any place(s) along the axis where the intensity is nearly zero? If so, find the point that is farthest from the aperture. If not, explain why not.

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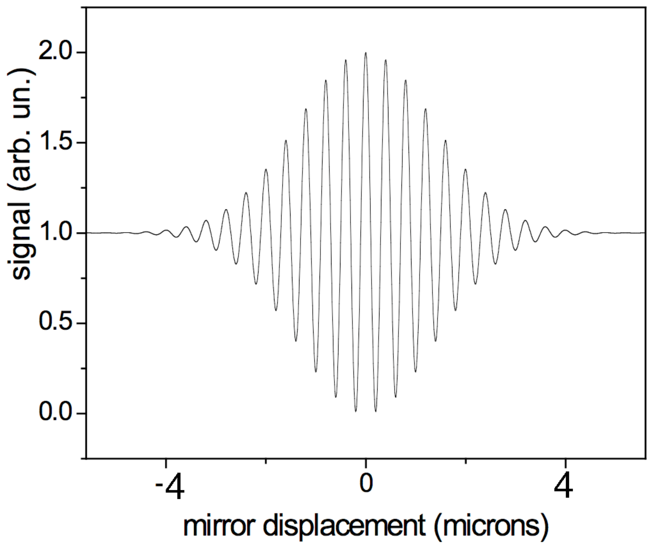
3. An astronaut is looking at a point source in a training pool (filled with water, n=1.33). The source is 200 cm from the vertex of the transparent hemispherical shield in front of the helmet, Where is the image of the point source that the astronaut sees? (The shield has a radius of 20 cm and is thin and of uniform thickness.)

4. An interferogram recorded with a Michelson interferometer is shown in the Figure. Estimate the wavelength, frequency, spectral width, and coherence time of the source.



5.

Linearly polarized light (a plane wave of irradiance I_0) is incident on a sequence of two crossed polarizers, see figure. The pass direction of the first polarizer, P1, is at an angle of $\beta = 22.5°$ with respect to the polarization of the incident light.

a) You have two quarter wave plates (QWPs). Where should you place them and what should be their orientation to maximize the irradiance at the detector?

b) Now, in addition to the QWPs, you have one polarizer at your disposal. Where should you place the QWPs and the polarizer to obtain maximum irradiance at the detector? What is the maximum irradiance at the detector plane, in terms of I_0 ?

6.

a) In one sentence, describe what is meant by "blazing" a diffraction grating.

b) A reflection grating has 500 grooves/mm when viewed at normal incidence. At what angle(s) is 200 nm light reflected in 2nd order?

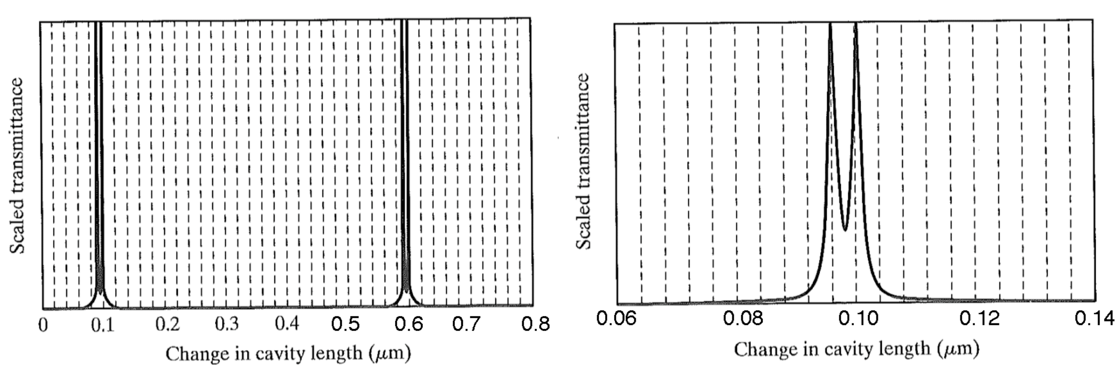
c) What blaze angle should be used to obtain maximum intensity in the 2nd order reflection for 200 nm light, with normal incidence?

7. A light source consisting of two components with closely spaced wavelengths is normally incident on a scanning Fabry-Perot interferometer with nominal length d = 2 cm. The scaled transmittance is shown in the Figure.

a) What is the nominal wavelength of the source?

b) Estimate the difference in wavelength of the two components, assuming they have the same mode number.

c) Estimate the difference in wavelength of the two components, assuming the right peak has a mode number one larger than that of the left peak.



8.

(a) A recent mission sought to launch a solar-sail powered spacecraft, which would have been pushed away from the sun by photons reflecting off the metallized sail.

Suppose the radiation force compensates for the gravitational force, so that the spacecraft stays at the same distance from the sun as the earth, but not orbit the sun, remaining stationary. For a 1000 kg ship, how large would the sail have to be?

It is 1.5 $\times 10^{11}$ m to the sun from the earth. The gravitational force at this distance from the sun is 6 Newton.

The sun radiates 3.9 $\times 10^{26}$ W.

(b) If the solar-sail spaceship were closer to the sun, the solar intensity would be higher, of course. Would the ship then accelerate away from the sun?

9. Answer 6 of the following 8 questions, in a couple of sentences each.

a) What is the physical meaning of phase velocity and group velocity?
b) Describe two methods for mode locking a laser.
c) Describe two methods for Q-switching a laser.
d) Explain the operation of an edge-emitting diode laser.
e) What waveplate could you use to change polarization from horizontal to vertical without loss? How would it be oriented?
f) Explain why Brewster windows are sometimes used in lasers.
g) Explain what a lock-in amplifier does and why it may be useful.
h) What should be the orientation of the transmission axis on polarized sunglasses? Why?

OSE 2013 Laser Exam

Answer all 3 questions. Begin each question on a new sheet of paper. Put your Banner ID at the top of each page. Staple all pages for each individual problem together.

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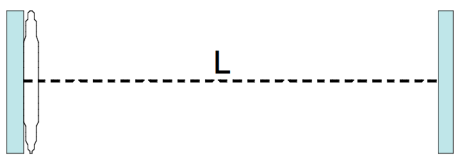
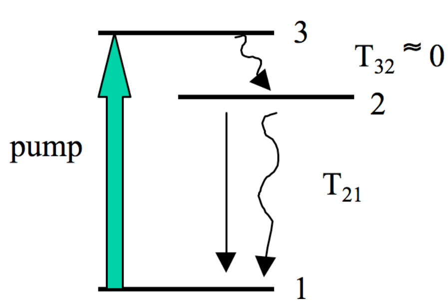
1 a) A laser resonator consists of two plane mirrors and one thin lens with focal length f=10 m in from of one of the mirrors. What is the radius of curvature of a mirror that would be equivalent to the lens – plane mirror sequence in the cavity? Explain your answer.

b) Calculate the position and size of the bean waist if the resonator length L=2 m and the laser wavelength $\lambda = 500$ nm.



2a) Write down the rate equations for the population number densities N_i [cm^{-3}] under lasing conditions assuming a certain pump rate R_p [$s^{-1}cm^{-3}$] and total number density N_t. For the active medium assume a three-level system, see diagram below. The homogeneously broadened laser transition is from level 2 to level 1 with cross section σ_{21} and is centered at frequency ν. For steady state, calculate the population inversion density $\Delta N_L = (N_2 - N_1)$. Neglect any dependence on the transverse (r) coordinate and ignore degeneracy factors.

b) Calculate the population inversion density ΔN_0 assuming there is no lasing. Sketch a graph showing $\Delta N_L/\Delta N_0$ as a function of the laser intensity.



3. A homogeneously broadened optical amplifier with a small-signal gain of 13 dB is irradiated with a wave with an intensity of 5 W/cm^2. The output intensity is 30 W/cm^2.

(a) Show that

$$\ln\frac{G}{G_0}+\frac{G-1}{I_s/I_{in}}=0$$

where $G_0=\exp(\gamma_0 L_g)$ is the small signal gain.

(b) What is the saturation intensity, I_s ?
(c) What is the maximum power (per unit area) extractable from this amplifier?

Note: G(dB)=10×log$_{10}$(G), where $G=I_{out}/I_{in}$