General Optics – 2014

Answer all questions; all count equally. Please begin each question on a new sheet of paper. Put your banner ID on the top of each page, and if you use more than one page for a question, staple the answer together.

1. a) In one sentence, describe what is meant by “blazing” a diffraction grating.

b) A reflection grating has 800 grooves/mm when viewed at normal incidence. At what angle(s) is 400 nm light reflected in 2^{nd} order?

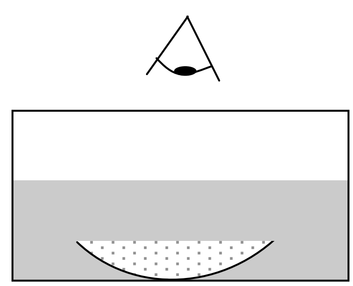
c) What blaze angle should be used to obtain maximum intensity in the 2^{nd} order reflection for 400 nm light, with normal incidence?

2. You are to build a microscope with 600X magnification overall, with 10X magnification in the eyepiece. The tube length (distance from objective to the intermediate image) is 30 cm.
a) Find the focal lengths of the objective and the eyepiece. (Recall that the virtual image an eyepiece makes is 25 cm behind the eyepiece.)

b) If the objective is to have a numerical aperture of 0.4, what must be its diameter?

c) What difficulties will you encounter if you build the microscope from simple (single element) lenses?

3. A plano-convex lens with a refractive index of 1.5 and power of 0.1 diopter (in air) is placed, convex surface down at bottom of a glass container (that is optically flat) filled with water (n = 1.33). Using a microscope and a sodium lamp (λ = 590 nm), we observe interference fringes from top. Determine the radius of the first dark ring.



4. An inexpensive quarter wave plate for 800 nm is to be made from quartz using $n_o = 1.544$ and $n_e = 1.553$. The nominal (approximate) thickness of the plate is 1 mm.

a) Ignoring dispersion, predict what this wave plate will do to the second harmonic at 400 nm?

b) How does your answer to (a) change given normal dispersion in quartz?

c) What is the effect of the waveplate on short pulses of light that have a center wavelength of 800 nm?

5. A Gaussian TEM_{00} beam coming out of a beam expander is used to illuminate a target at a distance L= 500 m. The beam has a waist at the output of the expander. If the beam has a wavelength of 500 nm and total power of 1 Watt: (a) find the waist (at the expander output) that guarantees the highest peak intensity on the target. (b) Calculate this intensity.

6. The output of a Michelson interferometer is fed to a photodetector.
a) The input is a (monochromatic) HeNe laser beam of wavelength λ. If one mirror translates at a speed of v, what is the modulation frequency f of the photocurrent?

b) Find a numerical valut for the modulation frequency f for a wavelength of 633 nm and translation speed of 5 mm/s.

c) The maximum mirror translation is 10 cm. What is the minimum resolvable optical frequency $\Delta\nu$ at 500 nm? [Note $\Delta\nu$ is an estimate of the smallest frequency separation of two monochromatic, mutually incoherent sources near 500 nm that the spectrometer is able to resolve.]

7. Answer any 5 of the following questions (i.e. you may skip 3 of them). Use no more than 150 words for each answer.

a) What is the physical meaning of phase velocity and group velocity?
b) Describe two methods for mode locking a laser
c) Describe two methods for Q-switching a laser
d) Why might your grandmother squint when threading a needle?
e) What waveplate could you use to change polarization from horizontal to vertical without loss? How should it be oriented?
f) Explain why Brewster windows are sometimes used in lasers.
g) What should be orientation of the transmission axis on polarized sunglasses? Why?
h) What is a Fresnel rhomb and what could it be used for?

8. An optical medium consists of N thin planar layers, stacked along the x-direction, and each of thickness Δx. The total thickness of the stack is $L = N\,\Delta x$, and the medium extends from $x = -L/2$ to $L/2$. The refractive index of each layer is known to vary due to the presence of an acoustic wave. While the refractive index of the medium in the absence of the acoustic wave is n, the change from layer to layer due to the presence of the acoustic wave is given by $\Delta n = 2\pi\Lambda^{-1}\,\delta_n\,\sin(2\pi\Lambda^{-1}x - \phi)\,\Delta x$, where Λ is the acoustic wavelength, ϕ is the instantaneous phase of the acoustic wave, and δ_n is a constant (determined by the amplitude of the sound wave, n and the photo-elastic constant of the material). The refractive indices of layers are related by $n_{k+1} = n_k - \Delta n$. An optical plane wave with frequency ν (and wave number $k = 2\pi n\nu/c_0$) is travelling in this medium, making an angle θ with each layer.

a) Let Δr_k be the incremental complex-amplitude reflectance associated with the change in the refractive index from the kth layer to the $(k+1)$th layer. Show that for TE and TM polarizations, Δr_k is approximately given by $\Delta r_k \approx -(1/[2n_k \sin^2\theta])\,\Delta n$. Make sure you state any key principles or equations that you use.

b) With some simplifying assumptions, one can add up the individual reflectances (taking phases into account) to show that the total complex reflectance amplitude r is proportional to $\mathrm{sinc}\{(2\pi\,\Lambda^{-1} - 2k\ \sin\theta)L/2\pi\}\exp(i\Omega t)$, where $\mathrm{sinc}(x)=(\sin\pi x)/\pi x$ and $\phi=\Omega t$, Ω being the frequency of the acoustic wave. Characterize the Bragg angle, θ_B, for this problem in terms of Ω and ν, and interpret it.

c) Describe a possible application of the phenomenon considered in this problem.

9. Consider a single-mode He-Ne laser emitting a 1 mW 633-nm $TEM_{0,0}$ Gaussian beam.

a) Calculate the average number of photons crossing the waist of the beam and within a time window of 100 ns?

b) Describe in detail a way to characterize theoretically the fluctuation in the actual number of photons relative to the mean number of photons found in Part (a).

c) What is the significance of the knowledge that the beam was generated by a He-Ne laser? How could your answer in Part (b) vary if we didn't have this information?

[Comment: not sure if we can expect them to know Poisson statistics.]

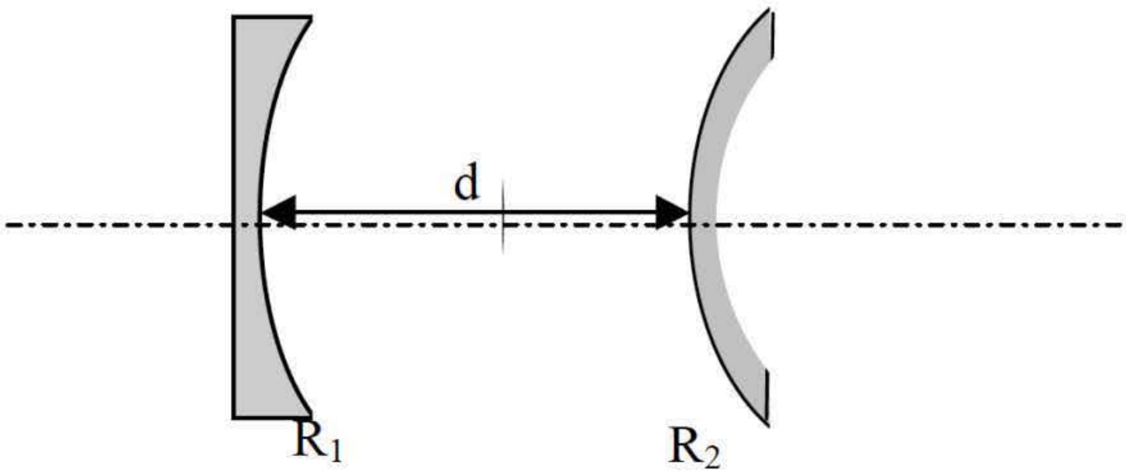
10. a) A laser, in a horizontal plane, has an output linearly-horizontal polarized (in that plane). Sketch a two mirror arrangement that would bring this beam in a higher horizontal plane, polarized vertically.

b) A left circularly polarized beam propagating along z is reflected back on itself.
What is the polarization of the reflected beam (propagating along -z)?
Use now a 90-45-45 degree prism as reflector to redirect the same beam from the z to -z direction. What is the polarization of the reflected beam (propagating along -z)?

[Comment: there is a phase shift for total internal reflection that makes this too difficult.]

Advanced Optics – 2014
Please begin each problem on a separate sheet of paper. Put your banner ID on each sheet. Do all problems. Some of the equations you need may be on the equation sheet and do not need to be rederived.

1. Consider the cavity shown below consisting of two curved mirrors separated by a distance d. $|R_1| = |R_2| = 50$ cm, and the wavelength of interest is $\lambda = 1$ μm.



a) A Gaussian laser beam circulates stably in this cavity when d = 30 cm. Find the location of the beam waist (which may be inside or outside the cavity). Qualitatively show your answer on the figure above.

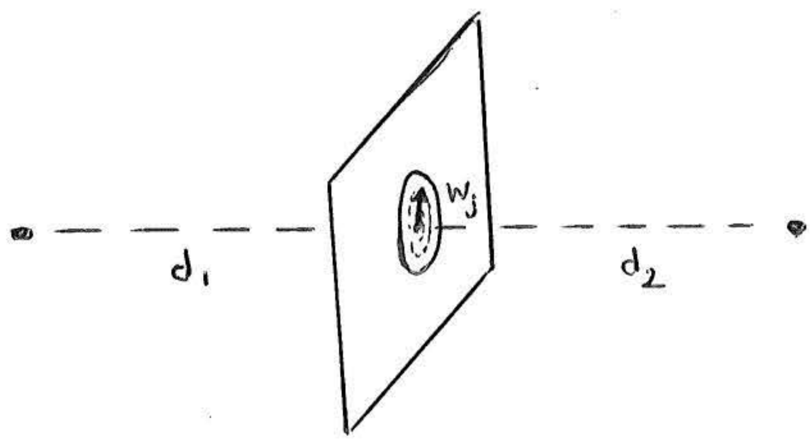
b) An external Gaussian laser beam is to be coupled into this cavity. This is done by matching the **q**-parameter (i.e. spot size and curvature) of the incident beam with that of the cavity mode. If the incident beam is coupled from the left (through mirror 1), what should be the incident beam spot size (w) and curvature (R) at the flat surface of mirror 1? Assume thin mirror substrates with index n=1.5.

2. Fresnel Lens.

a) Given a plane at a distance d_1 from a point source, and an observation point at a distance d_2 past the plane, derive an approximate expression for the radii W_j of the first few Fresnel zones. You may assume the radii are small compared with the d_1 and d_2.

b) Make a lens by allowing light to pass through the appropriate zones. Choose the 3 smallest zones that will work. Describe what zones you should use.

Show that for this Fresnel lens, the imaging formula $1/f = 1/d_1 + 1/d_2$ applies.



3. AO Modulator

Consider an acousto-optic modulator as creating a phase grating of transmission function:

$$1 + e^{-iKx}, \tag{1}$$

where K is the wavevector of a sinusoidal grating of period $\Lambda = 20\mu\text{m}$.

A plane optical wave of wave vector k_0 (wavelength in the modulator $\lambda_0 = 0.5\mu\text{m}$) is diffracted by this grating (see Fig. 1).

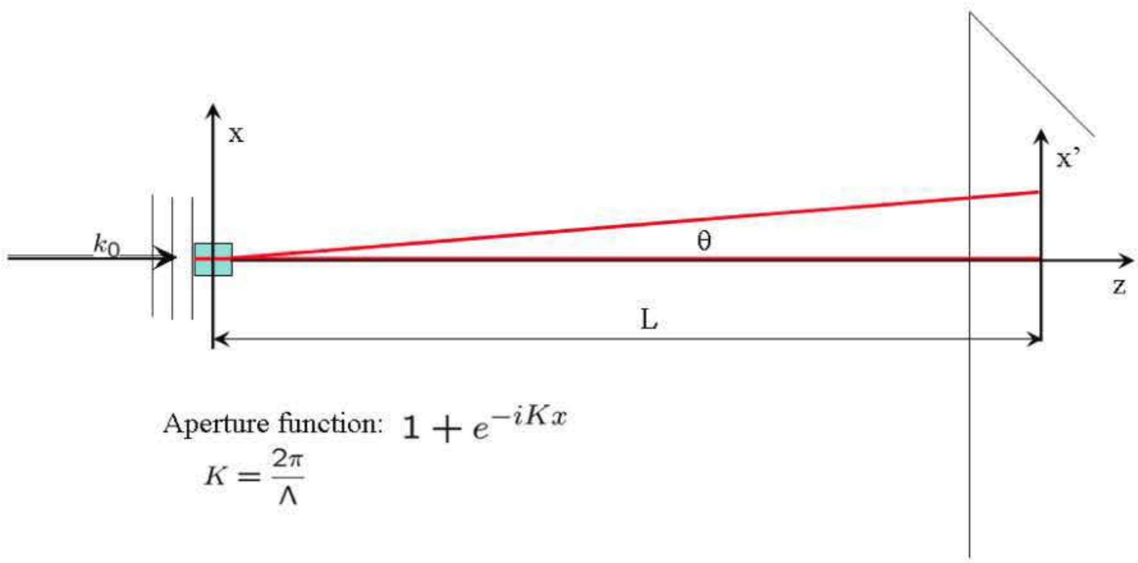


Figure 1: Beam diffraction by an acousto-optic modulator.

Question (a): What is the diffracted pattern?

(diffraction angle(s), spread of the diffraction angle, assuming that the phase grating is infinite along the x direction, and that the optical wave is an infinite plane wave).

Question (b): What is the diffracted pattern for an aperture of 100μm?

Assume here that the AO modulator has an aperture of only 100 μm. Find the complete diffracted field amplitude distribution $\tilde{\mathcal{E}}(\theta)$ [or alternately $\tilde{\mathcal{E}}(x')$ where $x' = L\theta$].

Possibly Useful Formulas

- Relation of spherical coordinates, (r, θ, ϕ), to Cartesian coordinates:

$$x = r\,\sin\theta\,\cos\phi, \quad y = r\,\sin\theta\,\sin\phi, \quad z = r\,\cos\theta.$$

Unit vectors:

$$\hat{r} = \sin\theta\,\cos\phi\,\hat{x} + \sin\theta\,\sin\phi\,\hat{y} + \cos\theta\,\hat{z};$$

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}; \quad \hat{\theta} = \hat{\phi} \times \hat{r}.$$

- Laplacian in spherical coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}.$$

- Magnetic field at position $\vec{r}$ due to a point magnetic dipole of moment $\vec{m}$ located at the origin:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}}{r^3}\right].$$

- Magnetization volume and surface current densities - $\vec{J}_M = \vec{\nabla} \times \vec{M}$, $\vec{K}_M = \vec{M} \times \hat{n}$.
- Force on a magentic dipole due to an external magnetic field: $\vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B})$.
- Force on a volume current distribution: $\vec{F} = \int \vec{J} \times \vec{B}\,d\tau$.
- Magnetic field, $\vec{B}$, of a uniformly magnetized sphere of magnetization $\vec{M}$ - inside it is uniform, equal to $(2/3)\mu_0\vec{M}$, and outside it is that due to an equivalent point magnetic dipole at the center.
- Time-averaged power radiated by an oscillating electric dipole:

$$P = \frac{\mu_0|p|^2\omega^4}{12\pi c}.$$

- Time-averaged power radiated by an oscillating magnetic dipole:

$$P = \frac{\mu_0|m|^2\omega^4}{12\pi c^3}.$$

- Magnetic field radiated by an oscillating magnetic dipole $\vec{m}\exp(-i\omega t)$ -

$$\vec{B} = \frac{\mu_0 k^2}{4\pi}(\hat{n} \times \vec{m}) \times \hat{n}\frac{\exp(ikr - i\omega t)}{r}, \quad k = \omega/c.$$

- Instantaneous power radiated by a non-relativistically moving charge with acceleration a (Larmor formula):

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}.$$

- The TM modes of a cylindrical waveguide with conducting walls have the following field relationships:

$$\vec{E}_T = \frac{ik}{\gamma^2}\vec{\nabla}_T E_z, \quad \vec{H} = \frac{\epsilon\omega}{k}\hat{z} \times \vec{E}_T,$$

where the subscript T on any vector denotes its transverse projection (i.e., projection in the xy plane), and k and γ are the longitudinal and transverse propagation constants of the mode,

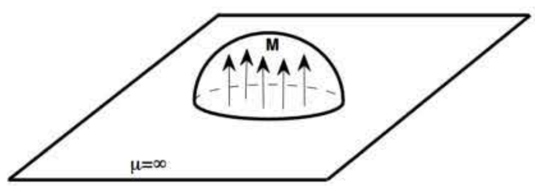
$$\gamma^2 + k^2 = (\omega^2/c^2)n^2, \quad n^2 = \frac{\epsilon\mu}{\epsilon_0\mu_0}.$$

For the TE modes, the above relation for $\vec{E}_T$ is replaced by a formally identical relation between $\vec{H}_T$ and H_z, while $\vec{E}_T = [(\mu\omega)/k]\vec{H}_T \times \hat{z}$.

Fall 2014
OSE Qualifying Examination
Classical Electrodynamics

Instructions: Solve any 3 problems. All problems carry equal points.

1. A solid hemispherical permanent magnet of radius R and uniform magnetization $\vec{M}$ oriented along its axis of symmetry is placed in contact with a flat infinitely permeable ($\mu = \infty$) plane located at $z = 0$, as shown.

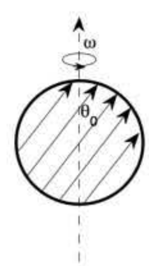


   (a) Use the method of images and the magnetic scalar potential to determine the magnetic field $\vec{B}(\vec{r})$ everywhere inside and outside the magnet for $z > 0$.

   (b) Determine the force with which the magnet attaches to the plane. (*Hint:* It would be simpler here to consider the magnetization currents, rather than the effective magnetic pole densities. You must use the arithmetic mean of the interior and exterior magnetic fields when computing the force on surface currents.)

   (c) How will your answers change if the infinitely permeable plane is replaced by an infinitely conducting plane? State your arguments carefully and quantitatively as far as you can without actually calculating the final expressions.

2. A good model for a pulsar is a magnetized sphere performing a rapid rotation about an axis through its center that is at some fixed angle with respect to its magnetization axis. Let the rotating sphere have a uniform magnetization $\vec{M}$, a small radius R, and angular velocity of rotation ω, while the angle between the rotation and magnetization axes is θ_0, as sketched below.

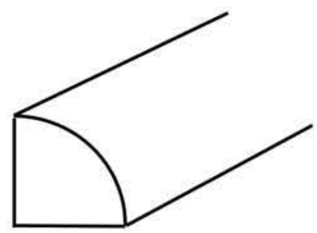


(a) Under what condition can the radiation from the sphere be regarded as pure magnetic dipole radiation?

(b) In the magnetic dipole limit, what is the total electromagnetic power radiated by the sphere?

(c) What is the polarization of the far-field radiation emitted along a direction in the equatorial plane of the rotating sphere? Along the polar axis of rotation?

3. An electromagnetic wave of frequency ω propagates inside a hollow metallic waveguide formed by two transverse planes and a cylindrical surface of radius R, as shown in cross-section in the figure.



(a) What types of waveguide modes are possible for this guide?

(b) For one type of modes, of your choosing, write down expressions for either the electric or the magnetic field inside the guide.

(c) Under what condition will only one of these modes will propagate in the guide? Express your answer in terms of clearly defined symbols.

(d) Determine the phase and group velocities for the modes, and show that their product equals c^2 for each of these modes.

Consider the following **instantaneous** representation of a standing wave:

$$E_x = \sqrt{3}E_0 \sin(\beta z)\cos(\omega t),\ H_y = -\sqrt{3}\cos(\beta z)\sin(\omega t).$$

As a function of z:

a) Calculate the instantaneous electric field energy density. [2 points]

b) Calculate the instantaneous magnetic field energy density. [2 points]

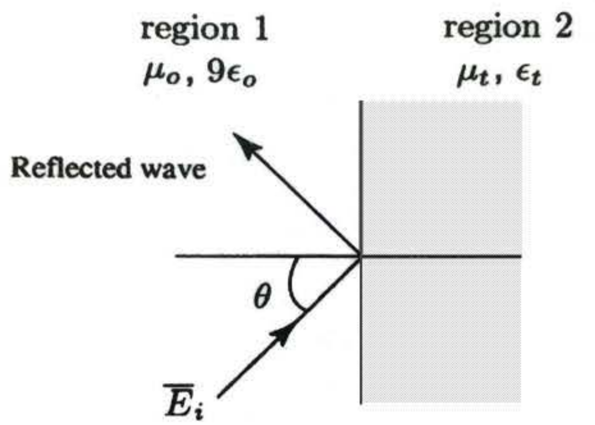
c) What is the relationship between the instantaneous electric field energy density and the instantaneous magnetic field energy density? Explain your answer. [2 points]

d) Calculate the instantaneous Poynting vector. [2 points]

e) Calculate the instantaneous **energy velocity**. (Use dimensional analysis to assist you in obtaining this expression if you do not remember it.) [2 points]

Consider a plane wave incident on a planar boundary at $z = 0$ from a dielectric medium with $\epsilon = 9\epsilon_0$ as shown in the figure below. The right-hand circularly polarized incident electric field is

$$\bar{E}_i = E_0\left[\left(\sqrt{3}\hat{x} + \hat{z}\right)cos(k_x x - k_z z - \omega t) + 2\hat{y}sin(k_x x - k_z z - \omega t)\right]$$



where E_0 is a real constant. The reflected field is

$$\bar{E}_r = E_0\left[R^{TM}\left(-\sqrt{3}\hat{x} + \hat{z}\right)cos(k_x x + k_z z - \omega t) + 2R^{TE}\hat{y}sin(k_x x + k_z z - \omega t)\right]$$

a) Show that the incident angle is 30^o. [3 points]

b) For $k_x = 1K_o$, find the frequency (Hz) and the wavelength (m) in region 1. (Note that $K_o = 2\pi$ rad/m, which is fundamental unit of spatial frequency and represents cycles per meter in spatial variation.) [3 points]

c) Find the value of ϵ_t ($0 < \epsilon_t/\epsilon_0 < \infty$) for which the incident angle is equal to the critical angle. Find the polarization of the reflected field. [2 points]

d) Find the value of ϵ_t ($0 < \epsilon_t/\epsilon_0 < \infty$) for which the reflected wave is linearly polarized. [2 points]

OSE Question #6

Consider a volume of electrons, of density n_0. Write the equation of motion of electrons under the influence of an electric field; neglect collisions and magnetic forces. Note that in the equation of motion of the electron, the electric field can be Coulomb field from the surrounding electrons.

Consider a perturbation δn_e from the equilibrium density of the electrons n_0. Show that the conservation equation for the electrons in terms of the density is

$$\left(\frac{\partial n}{\partial t} + \nabla n v = \text{Source terms}\right)$$

with $n = n_0 + \delta n_e$. Make the approximation of small v and $\delta n_e \ll n_0$.

The fluctuation of the density (position) electrons gives rise to an electric field. Consider that there is no other electric field (no applied field). Solve the equation of motion, conservation, and one of Maxwell's equation, to derive a differential equation for the density perturbation δn.

hint: both the velocity v and the (internal) field E can be eliminated among three equations.

Compare the equation in δn to the motion of an electric dipole. What are the similitudes? Is there a characteristic frequency?

Derive an equation for the (internal) electric field, using another Maxwell equation.

hint: the magnetic field is considered negligible and set to zero.

Indicate qualitatively how a collision term would modify the results.

Laser physics — 2014 Please begin each problem on a separate sheet of paper. Put your banner ID on each sheet. Do all problems. Some of the equations you need may be on the equation sheet and do not need to be re-derived.

1 Problem 1

(a) For the cavity sketched in Fig. 1, calculate the angles α and β that gives the maximal photon lifetime. This maximal lifetime is only for one polarization of light. What is that polarization?

(b) What is the maximum photon lifetime?

(c) What is the photon lifetime for the polarization orthogonal to the previous?

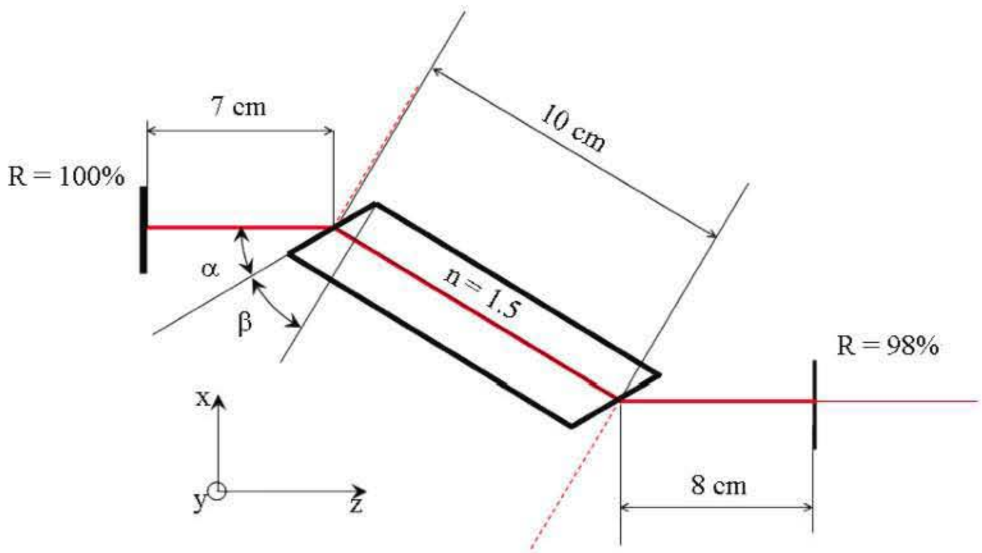
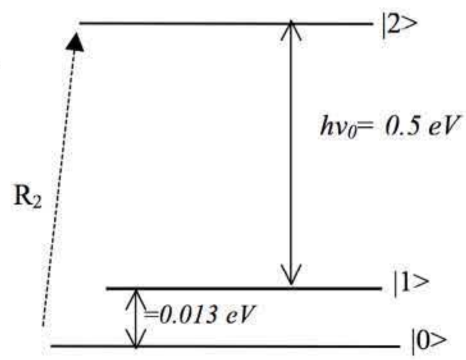


Figure 1: Cavity with laser rod.

2. A homogeneously (pressure) broadened gaseous system is described by the energy diagram below. The following parameters are known:

Dephasing time T_2 ($=1/g(\nu_0)$) = 10 ns
Spontaneous lifetime τ_{sp}=10 msec
No degeneracy: $g_2=1$, $g_1=1$, $g_0=1$, $n=1$
$N_{total}=N_0+N_1+N_2=10^{18}$ cm^{-3}
Temperature T=300 K

R_2 is the pump rate.



(a) What is the gain cross section at $\nu=\nu_0$?

(b) At thermal equilibrium, the population of state |2> may be taken as zero, while that in state |1> may not be. Explain why with a quick estimation.

(c) With R_2=0, and assuming thermal equilibrium, what is the absorption coefficient $\alpha(\nu_0)$? Note: $\alpha(\nu_0)= -\gamma(\nu_0)$, where γ is the small signal gain coefficient.

(d) Under steady-state conditions, calculate the number density N_2 required to make $\alpha(\nu_0)= -\gamma(\nu_0)=0$. Assume Boltzmann equilibrium between levels |0> and |1>. Note: it is not necessary to solve the rate equations to do this.

3. Explain 3 of the following devices or phenomena in a few sentences each. Do not exceed 200 words each.

- CO_2 laser
- Kerr lens mode-locking
- Semiconductor diode laser
- Pressure broadening
- Lifetime broadening
- Doppler broadening