Fall 2016
OSE Qualifying Examination
Classical Electrodynamics

Instructions: Solve any 3 of the 5 problems in the exam. All problems carry equal points. Also, you may replace the complex number i occurring in Problems 2 and 3 by $-j$ if you prefer the more conventional engineering notation.

Possibly Useful Formulas

- Relation of spherical coordinates, (r, θ, ϕ), to Cartesian coordinates:

$$x = r\,\sin\theta\,\cos\phi, \quad y = r\,\sin\theta\,\sin\phi, \quad z = r\,\cos\theta.$$

  Unit vectors:

$$\hat{r} = \sin\theta\,\cos\phi\,\hat{x} + \sin\theta\,\sin\phi\,\hat{y} + \cos\theta\,\hat{z};$$

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}; \quad \hat{\theta} = \hat{\phi}\times\hat{r}.$$

- Laplacian in spherical coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}.$$

- Azimuthally symmetric (ϕ-independent) solution of the Laplace equation in spherical polar coordinates:

$$V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell\,r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)\,P_\ell(\cos\theta),$$

  where the first few Legendre polynomials are defined as

$$P_0(\cos\theta) = 1; \quad P_1(\cos\theta) = \cos\theta; \quad P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1); \quad \text{etc.}$$

- Electric potential at position $\vec{r}$ due to a point electric dipole of moment $p\hat{z}$ located at the origin:

$$V(\vec{r}) = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3}.$$

- Polarization induced in a dielectric sphere of dielectric permittivity, ϵ, by a uniform external field, $\vec{E}$:

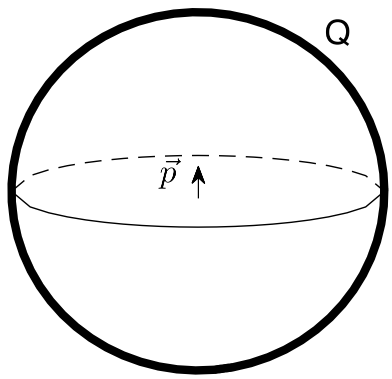
$$\vec{P} = 3\epsilon_0\left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right)\vec{E}, \quad \epsilon_r \equiv \frac{\epsilon}{\epsilon_0}.$$

- Force on an electric dipole due to an external electric field: $\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$.
- Force on a volume current distribution: $\vec{F} = \int \vec{J}\times\vec{B}\,d\tau$.
- Fresnel formulas for the amplitude reflection coefficient of a plane wave incident at a planar interface between two dielectrics:

$$r_\perp = \frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'}; \quad r_\parallel = \frac{n'\cos\theta - n\cos\theta'}{n'\cos\theta + n\cos\theta'},$$

  where $\perp, \parallel$ refer, respectively, to polarizations perpendicular and parallel to the plane of incidence. The angles of incidence and refraction are θ and θ', and n, n' are the refractive indices of the medium of incidence and the medium of transmission, respectively.

1. Consider a point electric dipole of moment $\vec{p}$ sitting at the center of a hollow conducting sphere of radius R and carrying a net charge of amount Q. The sphere sits on an insulating stand. By a proper choice of the coordinate system and in terms of the solution of the Laplace equation in spherical coordinates, involving the Legendre polyomials:



   (a) Calculate the potential everywhere inside the sphere.

   (b) Calculate the potential everywhere outside the sphere, including its surface.

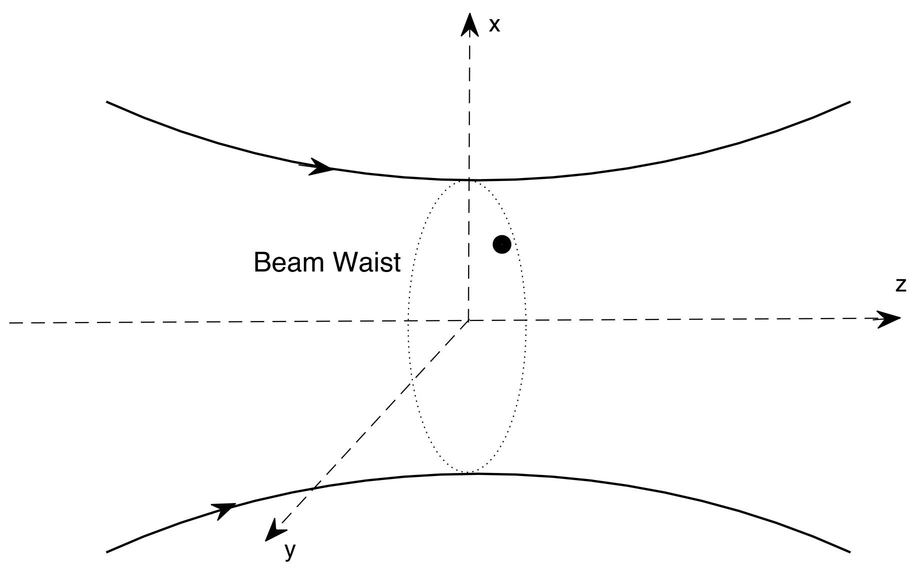
   (c) Calculate the surface charge density on the inner surface of the sphere.

   (d) Argue, without detailed calculation, why the total induced charge on the inner surface must vanish.

2. A small dielectric sphere of linear dielectric permittivity ϵ and radius a that is much smaller than the wavelength, λ, of a monochromatic Gaussian optical beam resides in the plane of its sharpest focus. Let the beam width, w, be large compared to the wavelength so the beam may be adequtely described as a transversely polarized beam. Let the beam be right circularly polarized. In its plane of sharpest focus, which we take to be the xy plane, the electric field of the beam traveling along the z axis has the following complex form:

$$\vec{E}(\vec{\rho}, z = 0, t) = (\hat{x} + i\hat{y})E_0 \exp(-\rho^2/w^2) \exp(-i\omega t),$$

where $\vec{\rho} = (x\hat{x} + y\hat{y})$ is the transverse position vector in the xy plane and the origin is taken to be at the center of the beam.

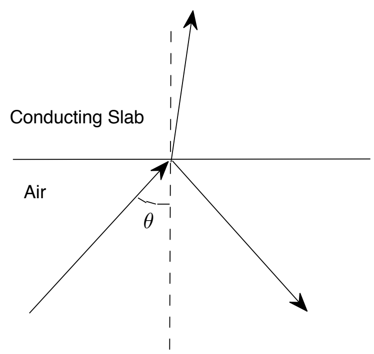


(a) If the sphere is placed off-beam-center at the point $(x_0, y_0, 0)$ in this plane, what time dependent polarization density will be induced in it in the small-radius approximation? (*Hint:* You can use, without deriving, the electrostatics of a dielectric sphere in a uniform electric field provided in the list of formulas to answer this question. Why is this adequate to calculate the spatial distribution of dielectric charges here?)

(b) What time-averaged electrical force does the beam apply on the sphere? Will it attract the sphere toward the beam center or repel it away from the beam center?

(c) Show that the time averaged electrical force on the sphere is radially inward. Due to this force, the sphere will perform simple harmonic oscillations in this plane, if released from rest from an off-axis position. If its mass is m, determine the frequency of oscillation.

3. A plane electromagnetic wave of angular frequency ω is incident at angle θ with respect to the normal of the surface of a semi-infinite slab occupying the region $z \geq 0$. Assume that the slab is highly conducting but non-magnetic, with a dielectric permittivity ϵ and conductivity σ that is large but not infinite, as shown in the figure.



   (a) Show from Maxwell-Ampere's law that the slab may be regarded as a dielectric with a complex effective permittivity equal to $\epsilon + i\sigma/\omega \approx i\sigma/\omega$.

   (b) Show that the wave transmitted into the conductor is an evanescent plane wave. What is the characteristic depth, in terms of σ, ω, and certain electromagnetic constants, to which the transmitted field propagates inside the slab in the high-conductivity limit, $\sigma >> \omega\epsilon$?

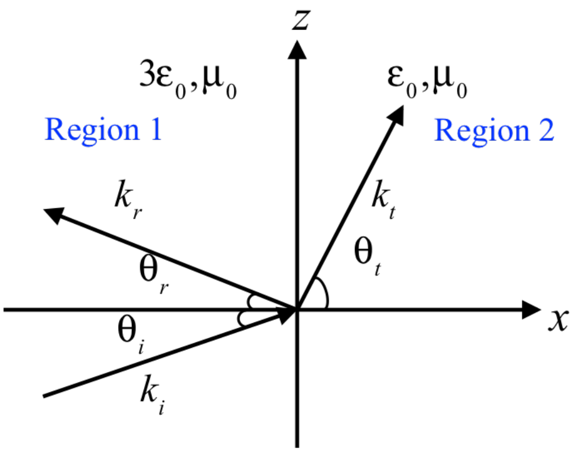
   (c) Show that in the high conductivity limit, the transmitted wave propagates essentially normally to the surface, regardless of the angle of incidence. Express n in terms of σ, ω and electromagnetic constants.

Surface Resistance [10 points]

What is the incident power density and the power absorbed per unit area in a sheet of brass $(\sigma = 1.5 \times 10^7 \text{ mho/m})$ onto which a uniform plane wave is incident with a peak electric field of 1.0 V/cm at 10.0 GHz. (Hint, the surface resistance is given by $R_s = \frac{1}{\sigma\delta}$ where σ is the conductivity and δ is the skin depth.)

Reflection and transmission [10 points]

Consider a plane wave incident from a dielectric region with permittivity $\varepsilon = 3\varepsilon_0$ upon a half space with $\varepsilon = \varepsilon_0$ as shown in the figure below.



a) Find the Brewster angle for Region 1 **[2 points]**.

b) Suppose the transmitted electric field is given by

$$\mathbf{E}_t = \hat{\mathbf{y}}\frac{E_0}{\sqrt{2}}\cos(k_{tx}x + k_{tz}z - \omega t) + E_0\frac{\hat{\mathbf{z}} - \hat{\mathbf{x}}\sqrt{3}}{2\sqrt{2}}\sin(k_{tx}x + k_{tz}z - \omega t)$$

i) Determine the incident and transmitted angles, θ_i and θ_t **[2 points]**

ii) What is the polarization of the transmitted field? Be sure to specify the handedness (left or right) if necessary **[2 points]**

iii) What is the polarization of the reflected wave? Be sure to specify the handedness (left or right) if necessary **[2 points]**

iv) Give an expression for the incident electric field, $\mathbf{E}_i$, and the reflected electric field $\mathbf{E}_r$ **[2 points]**

OSE Qualifying Examination - General Optics 2016
Answer all questions. Each question counts equally. Begin each question on a new sheet of paper. Put your Banner ID on each page.

You may find the formula for Doppler shift useful: $\frac{\lambda'}{\lambda} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$.

1a. Astronauts often train for weightlessness underwater, as shown in the figure.
Question: Why does this astronaut have such an apparently small head?
His visor is a thin hemispherical plastic shell of uniform thickness. There is air in his suit (of course) and water (n=1.33) in the pool. Use a ray tracing diagram to explain.

b. In the image, you can see a dark circle with his face in it. What is the ratio of the radius of this dark circle to the radius of the hemispherical visor? (Do not measure it, calculate it. You may assume the image was taken from far away.)

c. The camera taking this photo was 3 m from the astronaut. The visor has a radius of 16.5 cm. How far from him does the camera appear to be to the astronaut?

2a. Solar Sail. A perfectly reflecting sail is designed to hold a spaceship stationary at the same distance from the sun as the earth is from the sun (but far away from the earth.) The idea is to use the momentum transfer from solar photons reflecting off the sail (oriented perpendicular to the sun's radiation) to push the spaceship outward, countering the gravitational pull inward.

For a 1000-kg spaceship, how large does the sail have to be? It is 1.5 x 10^{11} m to the sun. The gravitational pull on the spaceship is 6 N. The sun radiates 3.9 x 10^{26} W.

b. If, instead, you wanted the spaceship to remain stationary at the distance from the sun that Mars is from the sun, would you need the sail to be larger, smaller, or the same size?

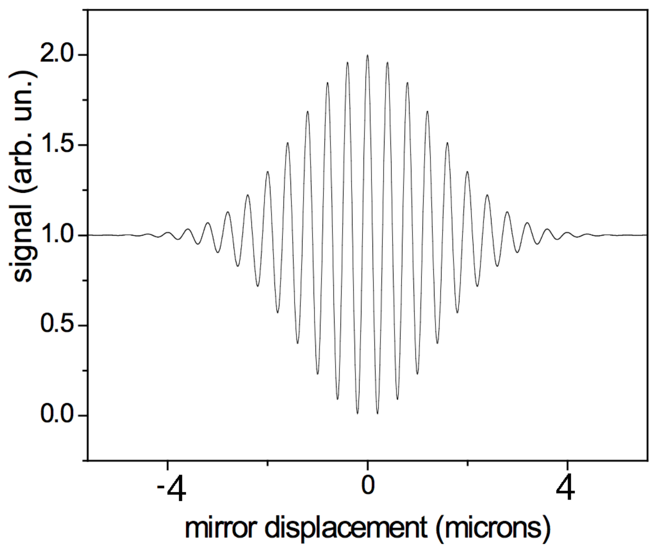
c. In part a, the sail is made 0.1% too large, so the spaceship begins to accelerate away from the sun. Describe its motion: does it accelerate forever away from the sun, eventually fall back toward the sun, or does it reach a steady terminal velocity? If so, what velocity?

3a. You wish to rotate the polarization of a monochromatic, collimated linearly polarized CW laser beam by 40°. What single optical element could you use, and how would you orient it?

b. If, instead, the laser were a low power short pulse laser, would you encounter problems? Would you need to specify a different, or better, optical element? If so, describe it.

c. Suppose that, in part a, the laser beam is diverging with a divergence half-angle of 30°. Does this have an effect on the polarization of the final beam? If the final beam is *not* linearly polarized, *estimate* *very roughly* what fraction of the intensity would pass a perfect linear polarizer oriented at 40°. (There's no formula to use… you need to explain how you made your very rough estimate!)

4a. An interferogram recorded with a Michelson interferometer is shown in the Figure. Estimate the wavelength, frequency, spectral width, and coherence time of the source.



b. Can you tell from the interferogram whether the source is CW or pulsed?

5a. Make a rough estimate of the distance from which car headlights can be resolved by the human eye. Assume the headlights are 1 m apart.

b. Suppose you want to distinguish a car from a motorcycle from 16 km away. Your plan is to use a Young's double slit experiment, so that the fringes from one car headlight cancel out the fringes from the other. (You'll put in a color filter to use only the yellow light from the headlights.) How far apart must the slits be?

c. For your headlight detector, the screen on which the interference fringes are to be observed is 1 m behind the slits. Your plan: if you see persistent fringes on the screen, the source is a motorcycle. If the fringes disappear when the object is 16 km away, it's a car. Do you anticipate any difficulties with your experimental plan? If so, what are they?

d. Your eyes are considerably farther apart than the size of your pupils. If you had one eye that was as large as your head, you could resolve headlights at a much greater distance. (Obviously, this capability was not terribly important in human evolution.)

Given that your retina only records light intensity, and not phase: is there any way to combine the signals from **both** eyes (using a novel bionic neuro-implant) so that you can determine whether a sub-resolution source is a single headlight or a pair?
If not, explain why in a sentence. If so, explain the general approach in a sentence.

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6. You are to build a microscope with 600X magnification overall, with 10X magnification in the eyepiece. The tube length (distance from objective to the intermediate image) is 30 cm.
a) Find the focal lengths of the objective and the eyepiece. (Recall that the virtual image an eyepiece makes is 25 cm behind the eyepiece.)

b) If the objective is to have a numerical aperture of 0.4, what must be its diameter?

c) What difficulties will you encounter if you build the microscope from simple (single element) lenses?

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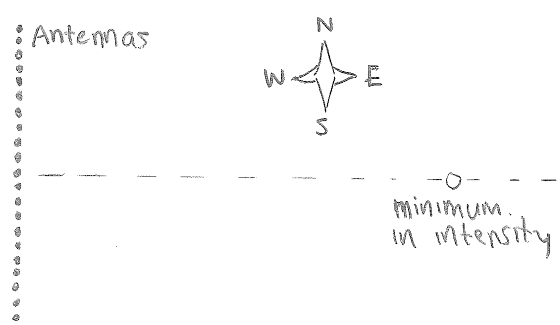
7. Consider 100 radio antennas arranged in a north-south line, separated by 10 meters each. Each antenna is broadcasting a vertically polarized sinusoidal wave at 6 MHz (in a dipole radiation pattern, i.e. isotropic in north, south, east, and west.) Very far to the east (say, 100 km), the signal from each antenna has an amplitude of 0.2 nV/m and an intensity I_1.

a. If the antennas are all emitting in phase, what is the resultant amplitude at this point (100 km to the east)? What is the intensity, in terms of I_1?

b. If the antennas are all emitting with random phases, what is the resultant amplitude at this point (100 km to the east)? What is the intensity, in terms of I_1?

c. Let's return to having the antennas all emitting in phase. As you move closer to the array from the east, (moving westward from 100 km away), you reach a point where the intensity goes through a minimum. *About* how far away from the antenna array is this point? (One approach to this problem is to contemplate a phasor diagram.)



8a. Sunlight reflects off an oil slick (a thin layer of oil on water) at normal incidence. The oil layer is 0.3 microns thick. The index of refraction of oil is 1.46; of water is 1.33. What visible color is seen in the reflection?

b. If the reflection is at 60° to the normal, what visible color is seen?

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9. A pulsed laser emits 10 ns pulses. The pulses are detected with a photodiode, which is connected to an oscilloscope. Next to the input of the oscilloscope is a switch. In one position, it says 50 Ω 13 pF. In the other position, it says 1 MΩ 13 pF. Make a rough sketch showing what the pulse would look like with each setting. Note any important differences between the observed signals.

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10. Answer each of the following in a sentence or two (each.)
a) Describe a method for mode locking a laser
b) Describe a method for Q-switching a laser
c) Why might your grandmother squint when threading a needle.
d) Explain why Brewster windows are sometimes used in lasers.
e) What should be orientation of the transmission axis on polarized sunglasses? Why?
f) What is a Fresnel rhomb and what could it be used for?

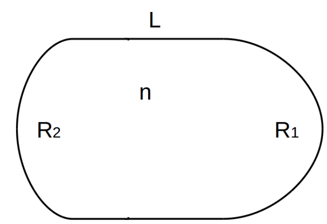
Advanced Optics: PhD Qualifying Examination – August, 2016.

Optical Sciences Concentration. Time allowed: 90 minutes

Answer all questions. All count equally. Begin each question on a new sheet of paper, and staple all pages for each question together. Put your banner ID# at the top of each page.

Q1: Consider a Michelson interferometer placed in water with refractive index of n_0=1.3 using a laser at λ=633nm. When the refractive index of the medium is raised by 0.0001, a bright fringe band shifts to where its adjacent dark band had previously been.
Calculate the difference between the lengths of the arms in the Michelson interferometer.

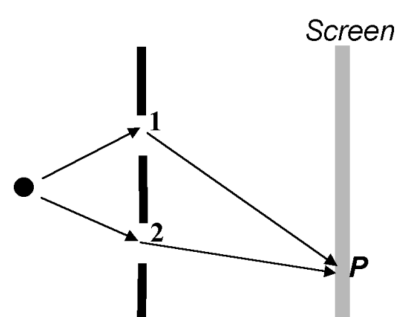
Q2: Quartz is a positive uniaxial crystal with n_e=1.553 and n_o=1.544. **(a)** Determine the retardation per mm at λ_0=633nm when the crystal is oriented such that retardation is maximized. **(b)** At what thickness(es) does the crystal act as a quarter-wave retarder?



Q3: Consider the biconvex thick lens shown in the adjacent figure, made from transparent material with index n and thickness L. **(a)** Derive the transfer ray matrix for this lens. **(b)** In terms of the radii of curvature, and n, obtain the thickness L for which the lens will act as a telescope for paraxial rays. (In other words, you want parallel entering rays to exit parallel). Write the ray matrix for this case. **(c)** If the telescope is used as a beam expander to double the diameter of a collimated beam, what condition must be satisfied by R_1 and R_2?

Q4: A partially coherent point source is illuminating a double-slit. **(a)** Show that the irradiance at an arbitrary point on the screen can be written as:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ Re[\gamma(\tau)], \quad \text{where} \quad \gamma(\tau) = \frac{\epsilon_0 c}{2} \frac{\langle E(t)E^*(t-\tau)\rangle}{\sqrt{I_1 I_2}}.$$



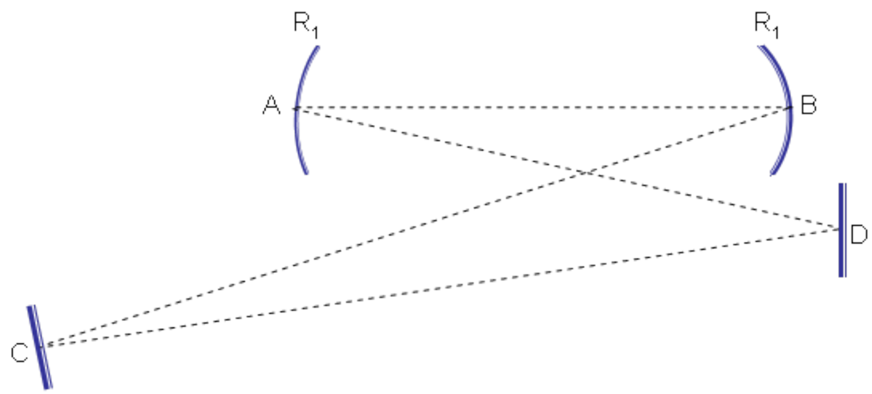
τ is the delay time associated with the difference between the optical path lengths from each hole to the observation point (P), I_1 and I_2 are the irradiances at point P when only one of the slits is open. $E(t)$ is the optical field generated by the point source at P when only one slit is open; assume equal irradiance at P from each hole. **(b)** Assuming $\gamma(\tau) = (1-\tau/\tau_c)e^{i\omega\tau}, \tau < \tau_c$; 0 for $\tau > \tau_c$ and equal irradiance from both holes show that the visibility of the fringes in the m'th order is given by: $V = 1 - (m\Delta\lambda/\lambda)$ where $\Delta\lambda$ is the line width of the point source.

OSE PhD Qualifying Exam- 2016

Lasers

Answer all questions. Begin each question on a new sheet of paper. Put your Banner ID# at the top of each page. Staple all pages for each individual question together (but do not staple the entire exam together.)

1. Consider the ring cavity shown below consisting of two curved mirrors (of same radius of curvature R_1) and two flat mirrors. Dimensions are: AB=12cm, BC=25cm, CD=25cm, and AD=20cm.



(a) How many beamwaists are in this cavity and where are the locations? (You do not need calculations to answer this question). *2 points*

(b) Obtain the ABCD matrix for this cavity. Clearly state your starting point and direction of rays. Assume the angle of incidence on the curved mirrors is small and will not cause any aberrations. *2 points*

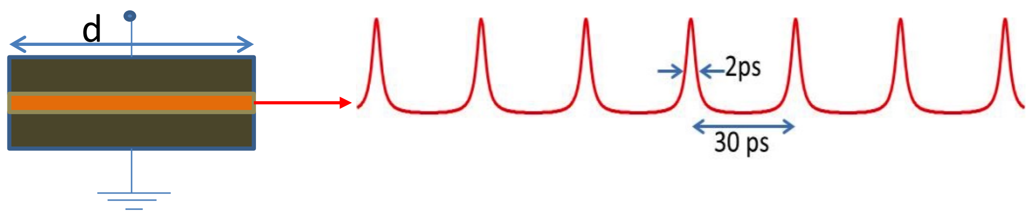
(c) Describe the procedure for calculating the Gaussian beam parameters (at any given point) in this cavity? *2 points*

(d) What is the range of R_1 for which this cavity is stable? *2 points*

(e) Briefly discuss the effect of oblique incidence (on the curved mirrors) on the beam parameters. *2 points*

OSE PhD Qualifying Exam- 2016
Lasers

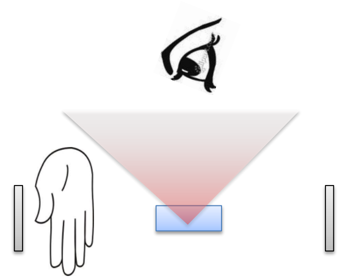
2. An edge-emitting diode laser is mode-locked (e.g. by gain modulation), outputting a pulse train consisting of 2ps (bandwidth limited) pulses separated by 30 ps (as shown). The refractive index of the gain medium is n=4.



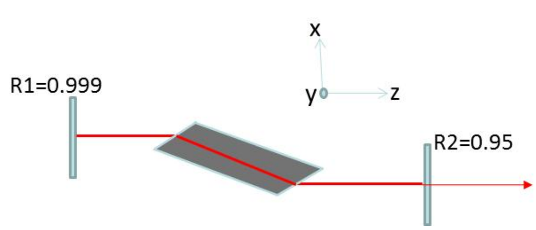
(a) What is the length (d) of the diode laser? (2.5 points)

(b) Qualitatively graph the power spectrum of the pulse train indicating the number of longitudinal modes that are lasing. (Be quantitative for the frequency axis; assume λ_0=1.5 μm). (2.5 points)

(c) A CW laser (in the visible) is operating at 5 times its saturation power. The graduate student working with this laser suddenly blocks one of the cavity mirrors and notices that the lasing stops (obviously!). At the same time, however, she also notices that the fluorescence viewed by her from the side of the laser crystal dramatically increases. Explain this phenomenon and quantify the amount of increase. (2.5 points)



(d) The solid state laser (shown below) is pumped 10× above threshold. If the saturation power of the gain medium inside this cavity is known (P_s=5W), estimate the output CW power. What is the polarization of the output (x,y or z or a combination)? (2.5 points)



Ray tracing matrices for $\begin{bmatrix} y \\ \alpha \end{bmatrix}$

Free space: $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ Lens $\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$ Curved Interface $n_1 - -n_2$: $\begin{pmatrix} 1 & 0 \\ \frac{n_1-n_2}{Rn_2} & \frac{n_1}{n_2} \end{pmatrix}$

Snell's law: $n_i \sin\theta_i = n_t \sin\theta_t$ Refraction at spherical interface: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2-n_1}{R}$

Grating equation: $\sin\theta' - \sin\theta = m\frac{\lambda}{d}$ Lens-makers's formula: $\frac{1}{f} = (\frac{n_2-n_1}{n_1})\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Gaussian beams

$$\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi n w^2} = \frac{1}{R} - \frac{i}{\rho}$$

$\rho = \rho_0\left(1 + \frac{z^2}{\rho_0^2}\right)$ $R = z\left(1 + \frac{\rho_0^2}{z^2}\right)$ $\rho_0 = \frac{\pi n w_0^2}{\lambda}$ $q_2 = \frac{Aq_1+B}{Cq_1+D}$

Phase transformation of a spherical lens: $\phi(x,y) = \frac{k}{2f}(x^2+y^2)$

$$\tilde{\mathcal{E}}_{mp}(x,y,z) = \mathcal{E}_0 H_m(\frac{\sqrt{2}x}{w})H_p(\frac{\sqrt{2}x}{w})\frac{w_0}{w}e^{-i\frac{kr^2}{2q}}e^{-i[kz-(1+m+p)\arctan z/\rho_0]}$$

For a cavity: $\frac{1}{q} = \frac{D-A}{2B} \mp \frac{i}{2B}\sqrt{4-(A+D)^2}$

Fresnel equations

$$r_\parallel = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i} \qquad r_\perp = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$$

$$t_\parallel = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} \qquad t_\perp = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$

Fabry-Perot *Field* transmission:

$$\mathcal{T} = \frac{(1-R)e^{-i\vec{k}\cdot\vec{d}}}{1-Re^{i\delta}} \qquad \delta = 2\varphi_r - 2\vec{k}\cdot\vec{d} = 2\varphi_r - 2kd\cos\theta |\mathcal{T}|^2 = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

Finesse $F = \frac{\pi\sqrt{R}}{1-R}$ Photon lifetime: $\tau = \frac{\tau_{rt}}{\delta_c} = \frac{Q}{\omega}$ $\delta_c = \Sigma$ gain/losses $= 2\alpha L + \ln\left(\frac{1}{R_1R_2}\right)$

Fringe visibility $V = \frac{2\sqrt{I_1I_2}|\gamma(\tau)|}{(I_1+I_2)}$ **Irradiance** $I = <S> = \frac{1}{2\sqrt{\mu_0/\epsilon}}\mathcal{E}_0^2 = \frac{nc\epsilon_0}{2}\mathcal{E}_0^2$

Faraday cell polarization rotation $\beta = VBd$ **Kerr electrooptic birefringence** $\Delta n = K\mathcal{E}^2\lambda$

Linear electrooptic effect $\Delta n = n^3\frac{r}{2}E$

Diffraction **diffraction integral** z = distance between aperture plane (ξ,η) and observation plane (x,y)

$$U_D(x,y) = \frac{ie^{-ikz}}{\lambda z}\int\int_{-\infty}^{\infty} U_A(\xi,\eta)e^{\left\{\frac{-ik}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]\right\}}d\xi x\eta$$

Fourier transforms $g(\Omega) = \int_{-\infty}^{\infty} f(t)e^{i\Omega t}d\Omega$ $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(\Omega)e^{-i\Omega t}d\Omega$

Blackbody radiation Energy density: $\rho(\nu)d\nu = \frac{8\pi h n^3\nu^3 d\nu}{c^3}\frac{1}{e^{h\nu/kT}-1}$ Power $P = \sigma A T^4$

Thermal equilibrium $N_2/N_1 = g_2/g_1\exp[-(E_2-E_1)/kT]$

Einstein coefficients $A_{21} = 1/T_1$ $B_{21} = \frac{c^3}{8\pi n^3 h\nu^3}A_{21}$ $g_2B_{21} = g_1B_{12}$

Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$ with $\int g(\nu)d\nu = 1$ Gain/loss (2-level): $\alpha_0 = \sigma\left[N_2 - \frac{g_2}{g_1}N_1\right]$

Gain (absorption): $\frac{dI}{dz} = (-)\frac{\alpha_0 I}{1+I/I_s}$ Integration: $\ln\frac{I(z,t)}{I_0(t)} + \frac{I(z,t)-I_0(t)}{I_s} = \alpha_0\ell$

Saturation energy density $W_s = h\nu/(2\sigma)$ Saturation intensity: $I_s = W_s/T_1$

Maxwell's equations

$$\begin{aligned}
\nabla \cdot \vec{D} &= \rho \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{H} &= J + \frac{\partial \vec{D}}{\partial t} \\
\nabla \cdot \vec{B} &= 0
\end{aligned}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

Curl identity: $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

Vector Potentials: $\vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$ $\quad \vec{E} = -\frac{1}{\epsilon} \nabla \times \vec{F}$ Complex Poynting vector $\vec{S} = \frac{1}{2}(\vec{E} \times \vec{H}^*)$

Biot Savart $dB = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \vec{x}}{|\vec{x}|^3}$

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ $\quad \vec{E} = -\nabla\phi$ Force on magnetic dipole $\vec{m}$: $\vec{F} = (\vec{m} \cdot \nabla)\vec{B}$ Torque: $\vec{\tau} = \vec{m} \times \vec{B}$

Magnetic induction $\vec{B}$ due to a magnetic dipole $\vec{m}$:

Near field:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{3(\vec{m} \cdot \vec{n})\vec{n} - \vec{m}}{r^3}, \quad \vec{n} = \frac{\vec{r}}{r}$$

Far field;

$$\vec{B} = \frac{k^2 \mu_0}{4\pi} \frac{e^{ikr}}{r} (\hat{n} \times \vec{m}) \times \hat{n}$$

Dipole Radiation

$\vec{H} = \frac{ck^2}{4\pi}(\hat{n} \times \vec{p}) \frac{e^{ikr}}{r}$

$\vec{E} = \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{H} \times \hat{n}$

Waveguides

$\vec{H}_t = \frac{\pm 1}{Z} \hat{z} \times \vec{E}_t$ $\quad Z = \left\{ \begin{array}{lll} \frac{k}{\epsilon\omega} & \text{(TM)} & \vec{E}_t = \pm \frac{ik}{\gamma^2} \nabla_t \psi \\ \frac{\mu\omega}{k} & \text{(TE)} & \vec{H}_t = \pm \frac{ik}{\gamma^2} \nabla_t \psi \end{array} \right\}$ $\quad (\nabla_t^2 + \gamma^2)\psi = 0$ $\quad \gamma^2 = \mu\epsilon\omega^2 - k^2$

where $\psi e^{\pm ikz}$ is $E_z(H_z)$ for TM(TE) waves

Physical constants $e = 1.6 \cdot 10^{-19}$ C

$\hbar = 6.6 \cdot 10^{-34}$ J · s

$c = 3 \cdot 10^8$m/s

$\hbar c = 1240$ eV · nm

$\epsilon_0 = 8.85 \cdot 10^{-12}$ F/m $\approx \frac{1}{36\pi} 10^{-9}$ F/m

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$k_B T = 26$ meV at T = 300 °K

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2}K^{-4}