GENERAL OPTICS

Answer all questions. Begin each question on a new sheet of paper and staple all pages together. Put your Banner ID on each page.

Question-1 (10 points)

Consider a 60-cm long HeNe laser (633 nm) that operates on two adjacent longitudinal modes of equal amplitude *A* and identical polarization. The laser illuminates a photodiode with a 1-ns response time. The diode output is monitored on a 1-GHz oscilloscope. Derive an equation for the signal as a function of time. Sketch the oscilloscope trace that you expect and comment.

Question-2 (20 points)

Describe qualitatively the following effects:

a) Pockels effect
b) Kerr effect
c) Faraday effect

Use sketches and simple equations as appropriate. Comment on whether each effect is a non-linear optical effect, the order of the effect; the properties (symmetry, etc.) that the material must have to exhibit each effect.

Question-3 (20 points)

(a) Fresnel (near-field) and Fraunhofer (far-field) diffraction theories are based on different assumptions regarding the variation in phase of light (at the observation point) originating from different points of the aperture. What are those assumptions?
(b) If one wishes to calculate the diffracted intensity 10 cm past at 0.1 mm diameter hole illuminated with plane waves of wavelength 1 micron, is far-field diffraction theory appropriate? Show your work (i.e. do not guess)
(c) Does your answer to (b) changes if wavelength is 1 mm?
(d) Does your answer to (b) changes if light is not a plane wave, but emitted from a point source 1 mm in front of aperture?

Question-4 (20 points)

a) In one sentence, describe what is meant by "blazing" a diffraction grating?
b) A reflection grating has 500 grooves/mm when viewed at normal incidence. At what angle(s) is 200 nm light reflected in 2^{nd} order?
c) What blaze angle should be used to obtain maximum intensity in the 2^{nd} order reflection for 200 nm light, with normal incidence?

Question-5 (10 points)

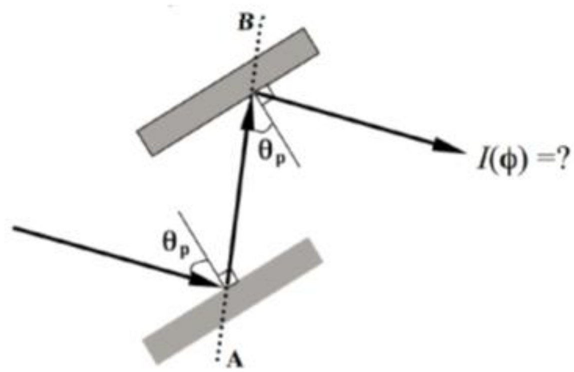
Candle and a screen are separated by 40 cm. When the lens is placed between them, there are two positions of the lens for which a sharp image is obtained on the screen. The lens positions which give sharp images are separated by 20 cm. What is the focal length of the lens? If the bottom half of the lens is covered, how will the image be changed (qualitatively)

Question-6 (10 points)

A mode locked laser emits a continuous train of 1 ps pulses 10 ns apart. Estimate the number of longitudinal modes that oscillate in the laser cavity.

Question-7 (10 points)

The following figure depicts an unpolarized ray of light reflecting off two parallel dielectric plates at the polarization angle θp (Brewster angle). If we rotate the upper plate around AB line through an angle ϕ so that the reflected ray comes out of the plane of the paper. Describe the irradiance of the emerging beam as a function of ϕ from 0 to 90 degree.



Question-8 (10 points)

a) What waveplate would you use to change the polarization of a beam from horizontal to vertical? How would you orient the waveplate?
b) A birefringent material has indices 1.500 and 1.502. What is the thinnest section of this material that can be used as a quarter wave plate for λ=500 nm? Is that practical? How can we make a mechanically sound quarter wave plate?
c) What intensity fraction of initially unpolarized light is transmitted by a quarter wave plate?

Question-9 (10 points)

Describe Q-switching and typical characteristic of a Q-switched laser output. Give one example each for both active and passive Q-switching techniques. Do not exceed one page.

Question-10 (10 points)

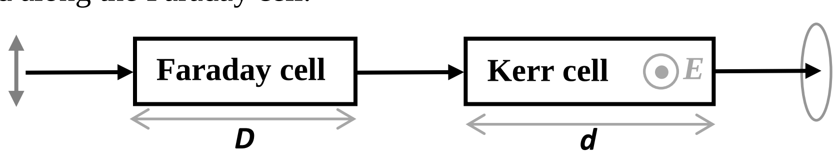
A laser emits 50 mW at a wavelength of 500 nm. If the beam is collimated to a diameter of 10 mm, estimate the maximum photo-current which could be generated if the beam uniformly irradiates a photodiode of diameter 1 mm.

Question-11 (10 points)

A binary star system in the constellation Orion has an angular interstellar separation of 10^{-5} rad. If λ=500 nm, what is the smallest diameter the telescope can have to just resolve the two stars?

Question-12 (10 points)

A Faraday and a Kerr modulator (with an electric field perpendicular to the figure plane) are used to transform a linearly polarized beam to a circularly polarized beam. Assuming the Verdet constant (V), half-wave voltage (V_{HW}) and the length of both modulators are given constants; find the voltage applied across the Kerr cell and the magnetic field applied along the Faraday cell.



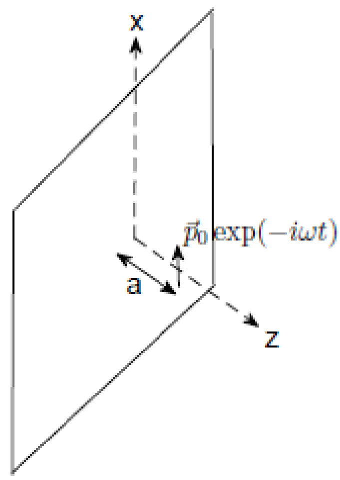
Instructions:

Solve any 3 of the 6 problems in the exam. All problems carry equal points.
Begin each question on a new sheet of paper and staple
all pages together. Put your Banner ID on each page.

You may replace the complex number ***i*** occurring in Problems 1, 2, and 3 by ***-j*** if you prefer the more conventional engineering notation.

ELECTROMAGNETISM

1. Consider an oscillating point electric dipole of moment $\vec{p}_0 \exp(-i\omega t)$ located on the z axis a distance a away from an infinite, conducting plane at $z = 0$.



   (a) Consider the special case of $\vec{p}_0$ being oriented along the x axis: $\vec{p}_0 = p_0\hat{x}$. Show that the boundary conditions on the electric ($\vec{E}$) and magnetic ($\vec{B}$) fields at the conducting plane are obeyed if the plane is replaced by an image dipole oscillating in phase as $\vec{p}' \exp(-i\omega t)$ at the image location. How are $\vec{p}'$ and $\vec{p}_0$ related?

   (b) What is the total oscillating $\vec{E}$ field at a point $\vec{r}$ along direction $\hat{r}$ a very large distance r away from the origin in the radiation zone?

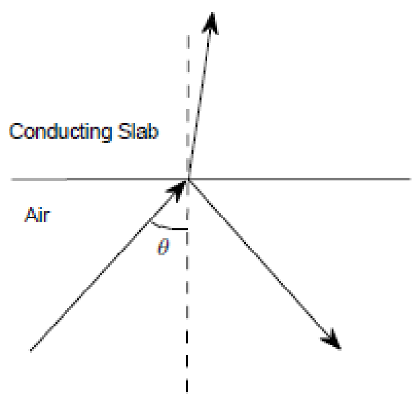
   (c) Consider this field for the case of $\vec{p}_0$ being oriented along the x axis. Show that for large enough a the radiation field shows an interference pattern of maxima and minima as a function of the angle θ that the observation vector $\hat{r}$ makes with the z axis. (*Hint:* The distances from the given and image dipoles to a point far away are different, with their difference easily calculated by means of simple geometry.)

   (d) What are the values of θ for which the radiation field vanishes?

   (e) How large does a have to be to yield at least one direction of complete destructive interference in the half space $z > 0$?

ELECTROMAGNETISM

2. A plane electromagnetic wave of angular frequency ω is incident at angle θ with respect to the normal of the surface of a semi-infinite slab occupying the region $z \geq 0$. Assume that the slab is highly conducting but non-magnetic, with a dieletric permittivity ϵ and conductivity σ that is large but not infinite, as shown in the figure.



(a) Show from Maxwell-Ampere's law that the slab may be regarded as a dielectric with a complex effective permittivity equal to $\epsilon + i\sigma/\omega \approx i\sigma/\omega$.

(b) Show that the wave transmitted into the conductor is an evanescent plane wave. What is the characteristic depth, in terms of σ, ω, and certain electromagnetic constants, to which the transmitted field propagates inside the slab in the high-conductivity limit, $\sigma >> \omega\epsilon$?

(c) Show that in the high conductivity limit, the transmitted wave propagates essentially normally to the surface, regardless of the angle of incidence.

(d) Using the Fresnel reflection formula, show that in the high-conductivity limit and for normal incidence, the fraction of the incident power reflected by the slab is $1 - 4\mathrm{Re}(n)/|n|^2$, where n is the effective index of refraction of the conducting slab? Express n in terms of σ, ω and electromagnetic constants.

(e) Use the result obtained in part (d) to calculate the fraction of incident power absorbed by the slab. Does your answer agree with the result that in the infinite-conductivity limit, no power is absorbed by the conductor?

ELECTROMAGNETISM

3. A transverse electromagnetic wave with complex amplitude $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})\exp(-i\omega t)$ travels through a neutral plasma in which the electrons may be consider essentially free particles.

   (a) Show using the Drüde-Lorentz model that under reasonable assumptions the current density has the expression

   $$\vec{J}(\vec{r},t) = \frac{iNe^2}{\omega m}\vec{E}(\vec{r})\exp(-i\omega t),$$

   where e is the electronic charge, m is the electronic mass, and N is the number density of electrons in the plasma.

   (b) Use Maxwell's equations to show that the electric field inside the plasma obeys the following wave equation:

   $$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\vec{E}(\vec{r}) = \frac{\omega_p^2}{c^2}\vec{E}(\vec{r}),$$

   where $\omega_p = \sqrt{Ne^2/(m\epsilon_0)}$ is the plasma frequency.

   (c) What is the index of refraction for a plane wave of frequency ω inside such a plasma?

Problem 4

a) What does this equation describe? Explain each term in the equation [(1)-(4)] and define each variable. **[5]**

$$\underbrace{-\frac{1}{2}\iiint_V (\mathbf{H}^*\cdot\mathbf{M}_i + \mathbf{E}\cdot\mathbf{J}_i^*)\,dv}_{(1)} = \underbrace{\oiint_S \left(\tfrac{1}{2}\mathbf{E}\times\mathbf{H}^*\right)\cdot d\mathbf{s}}_{(2)} + \underbrace{\tfrac{1}{2}\iiint_V \sigma|\mathbf{E}|^2\,dv}_{(3)} + \underbrace{j2\omega\iiint_V \left(\tfrac{1}{4}\mu|\mathbf{H}|^2 - \tfrac{1}{4}\varepsilon|\mathbf{E}|^2\right)dv}_{(4)}$$

b) From this equation, derive an expression for the total power radiated P_{rad} from an antenna located at the origin of the coordinate system and evaluated in the far-field. **[2]**

c) What distance from the antenna denotes the transition from the radiating near-field to the far-field? **[1]**

d) What is the difference between your answer to part b) when evaluating in the near-field as opposed to the far-field of the antenna? **[2]**

Problem 5

(a) Write the instantaneous forms of Maxwell's Equations **[2]**

(b) Write the SI units of each electromagnetic variable in Maxwell's Equations **[2]**

(c) Given the total electromagnetic energy

$$\mathrm{W} = \frac{1}{2}\int_{\boldsymbol{V}} (\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})dV$$

Derive from instantaneous forms of Maxwell's Equations that: **[4]**

$$\frac{\partial \mathrm{W}}{\partial \mathrm{t}} = -\oint_{S} (\mathbf{E}\times\mathbf{H})\cdot d\boldsymbol{S} - \int_{\boldsymbol{V}} \mathbf{E}\cdot\mathbf{J}\, dV$$

d) What are the physics significances of the surface integral and volume integral in Question (c)? **[2]**

Problem 6

A linearly x-polarized uniform plane wave with E-field amplitude 1 V/m propagating in air in the +y direction is normally incident on an infinite slab of mica. The frequency is f = 120 MHz. Mica has $\varepsilon_r = 5.0$, $\mu_r = 1.0$, and $\sigma = 0$.

a) Write an expression for the incident electric field vector, $\mathbf{E}^{\mathrm{i}}$, in the instantaneous form. **[2]**

b) Find the reflection and transmission coefficients. **[2]**

c) Find the time-average Poynting vectors in both regions. **[2]**

d) Find the incident, reflected, and transmitted power densities. **[2]**

e) Find the standing wave ratio in the air. **[2]**

ADVANCED OPTICS

Solve all problems. Begin each problem on a new sheet of paper, and staple all pages together. Put your banner ID# at the top of each page.

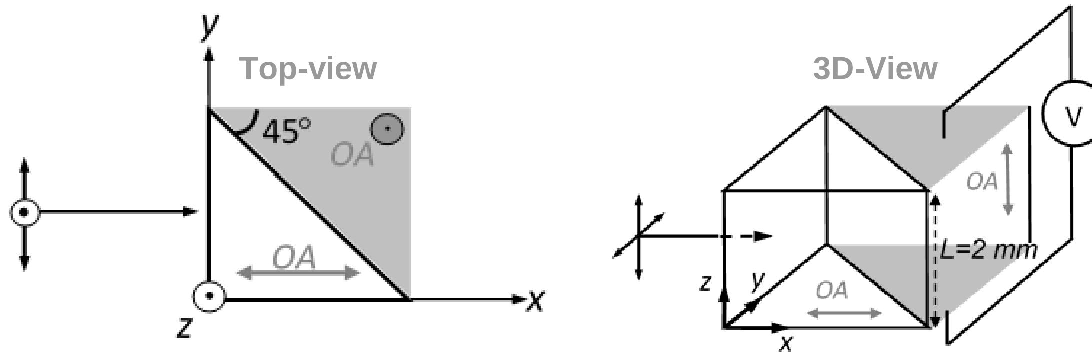
Problem #1 (40 points)

A beam of unpolarized light is normally incident on a polarization cube made of uni-axial electro-optic crystal ($n_o = 2$, $n_e = 2.2$). The electro-optic tensor of the crystal has only one nonzero element: r_{33}=100 pm/V. The cube consists of two sections shown below: a passive section (with optical-axis parallel to x-axis) and an active section (optical-axis parallel to z-axis).

a) Calculate the angle between the two beams that are *transmitted* through the cube when V= 0.

b) A DC voltage V=10 KV is applied on the active section using two parallel electrodes (gray sections). Calculate the refractive index for two polarization component (⊙ and ↔) in the active section?

c) When the voltage is applied, which one of the transmitted beams will be deflected and by how many degrees?



Problem #2 (30 points)

A Michelson interferometer (with 50/50 splitter) forms fringes with cadmium red light of 643.847 nm and linewidth of 0.0013 nm. If the complex degree of coherence of the source can be expressed as $\exp(-|\tau|/\tau_c)$ where τ_c is the coherence time of the source:

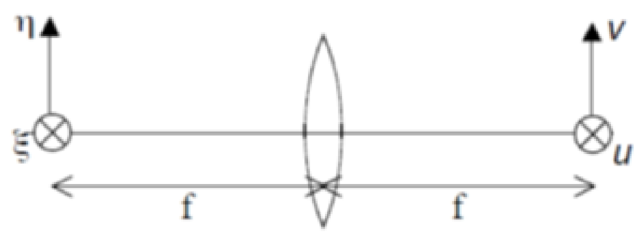
(a) What is the visibility of the measured fringes when one mirror is moved 5 cm from the position of zero path difference between arms?

(b) How much should we move one of the mirrors so that the visibility drops by 100 times?

(c) Using the same source, can we accurately measure the Finesse of a 1 m long Fabry-Perot cavity made of two identical mirrors with 95% reflectivity? (explain your answer)

Problem #3 (30 points)

A planar input transparency with amplitude transmittance $\boldsymbol{t_a(\xi,\eta)}$ is placed in front of the focal plane of a converging lens and is uniformly illuminated by a normally incident, monochromatic plane wave of amplitude $\boldsymbol{A}$. Using *Fresnel* diffraction derive the diffraction pattern ($\boldsymbol{U(u,v)}$) at the back focal plane of the lens and show it is exactly the Fourier transform of $\boldsymbol{t_A}$.



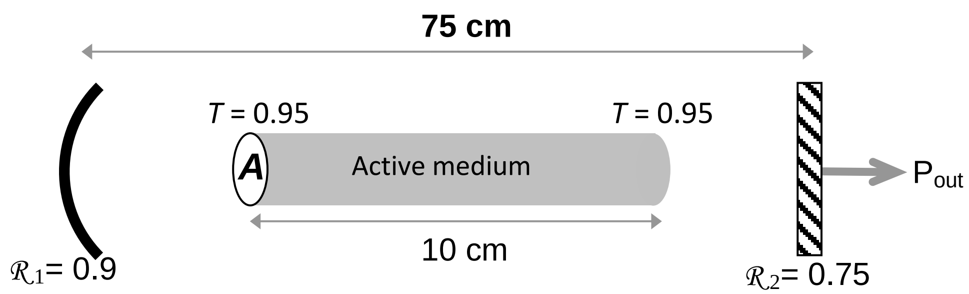
Hint: To simplify your caclulaitons use frequency domain propagation to propagte the field from the transparency to the lens (in frequency domain) and Fresnel integral for the rest.

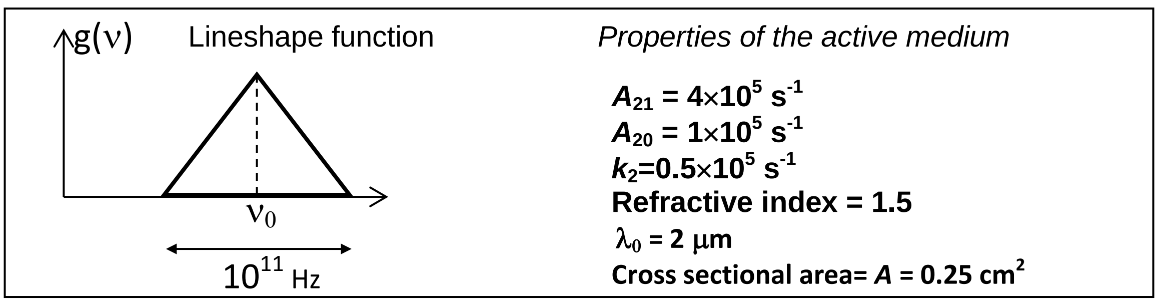
LASER PHYSICS

Solve all problems. Begin each problem on a new sheet of paper and staple all pages together. Put your banner ID# at the top of each page.

Problem 1 (33 points)

Consider the high-Q cavity and a 3-level active medium in the figure below:





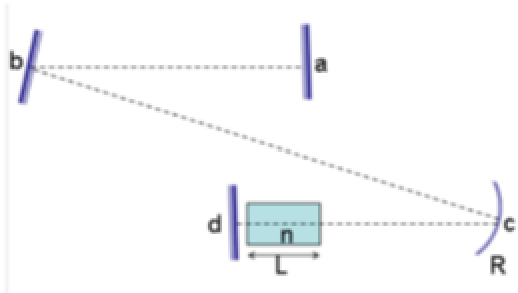
(a) Compute the threshold value of the population inversion (ΔN_{th}).

(b) We pump the medium such that its steady state small signal gain coefficient (γ_0)is $2\times\gamma_{th}$, calculate the output power (P_{out}).

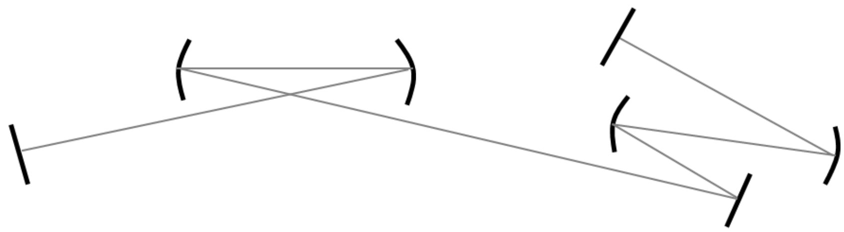
(c) If this laser is passively mode-locked estimate its *repetition rate*, pulse width and *peak output power*.

Problem 2) (34 points)

Consider the cavities shown below. Cavity #1 is a laser cavity consisting of a gain medium with a length L and refractive index n, three flat mirrors and one concave mirror with radius R. Cavity #2 is a passive cavity with four concave mirrors and three flat mirrors.



Cavity # 1



Cavity # 2

(a) Without any calculations, what are the *minimum* and *maximum* number of beam waists for each cavity? Identify their locations.

(b) ONLY for CAVITY#1: Assuming geometric distances **ab**=d_1, **bc**=d_2 and **cd**=d_3 are known, choose an appropriate starting point and setup the matrix product for evaluating the roundtrip ABCD matrix for this cavity (do not multiply the matrices).

(c) Assuming the ABCD matrix is known for each cavity and the fact that the beam form should be preserved after each roundtrip, <u>derive the stability condition for this cavity (only in terms of A, B, C, and D values)</u>

Problem 3) (33 points)

A homogeneously broadened optical amplifier with a small-signal gain of 13 dB is irradiated with a wave with an intensity of 5 W/cm^2. The output intensity is 30 W/cm^2.

a) Show that when $\nu=\nu_0$:

$$ln\frac{G}{G_0}+\frac{G-1}{I_s/I_{in}}=0$$

Where $G= I_{out}/I_{in}$ and $G_0=\exp(\gamma_0 L_g)$. L_g is the amplifier length

b) Calculate the saturation intensity $\boldsymbol{I_s}$.

c) What is the maximum power (per unit area) extractable from this amplifier?

Note: G(dB)=10×log$_{10}$$^{(G),}$ where $G=I_{out}/I_{in}$