Instructions:

Answer all problems

Begin each question on a new sheet of paper!

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1-You may remember the general rule linking the phase shift on reflection φ_r, and the phase shift on transmission:

$$\varphi_r - \varphi_t = \frac{\pi}{2}$$

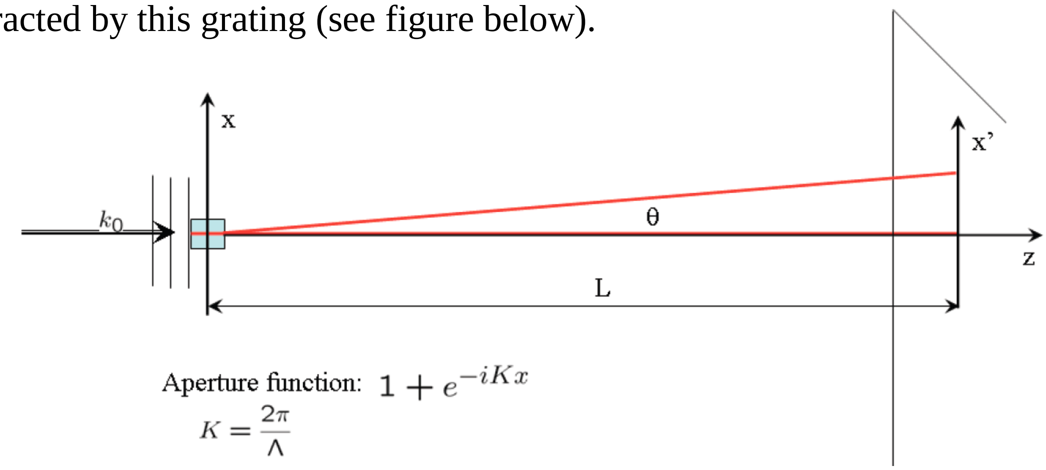
Which was derived from simple energy conservation. Verify this rule for a Fabry-Perot. (hint: you can prove this directly or make an approximation assuming the incident beam is slightly off resonance (e.g. $\delta=2N\pi+0.1$) and the reflectance of each mirror is 0.91).

(30 points)

2-Consider an acousto-optic modulator as creating a phase grating of transmission function:

$$1 + e^{-iKx}$$

where K is the wave vector of a sinusoidal grating of period Λ=20 μm. An infinite plane wave of wave vector k_0 (wavelength in the modulator λ_0 = 0.5 μm) is diffracted by this grating (see figure below).



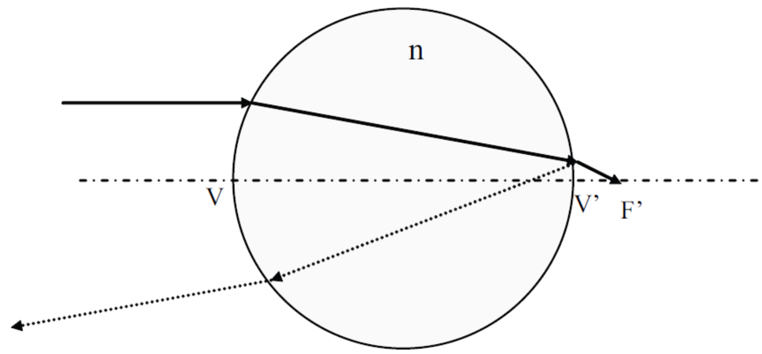
Find the complete diffracted field amplitude distribution $E(\theta)$ (or alternatively $E(x')$ where $x'=L\theta$):

a) Assuming the phase grating is infinite along the x direction.

b) Assuming the phase grating has an aperture of only 100 μm.

(35 points)

3-Consider a sphere having a radius R, an index n, and surrounded by air (n_{air}=1), as in figure below:



a) Derive an expression for the position of the image focal point (*F'*) with respect to V'.

b) Find out the index for which the reflected paraxial rays (shown by dashed lines) are parallel to the incident rays (retro-reflection). Show the ray paths.

c) Is there a refractive index for which the above sphere will act as a telescope (that is a collimated incident beam exits the sphere collimated)? Explain your result.

(*you can use the matrix approach or direct calculation based on Snell's law*)

(35 points)

Instructions:

Solve any 3 of the 5 problems in the exam.
All problems carry equal points.
On top of the first page write the number of problems that you have selected

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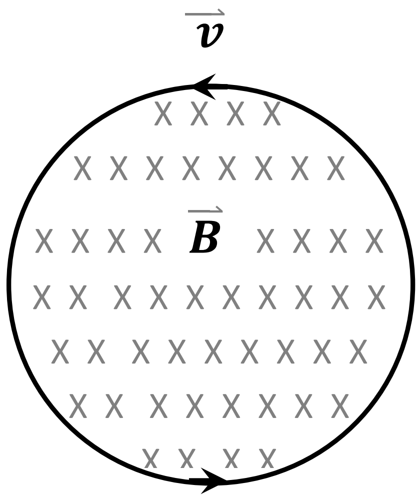
1-A point charge q of mass m moves on a circle of initial radius R under the Lorentz force exerted by a uniform static magnetic field $\boldsymbol{B}$ that is orthogonal to the plane of the circle. Neglect all other forces on the charge and take its motion to be non-relativistic.



(a) What is the speed of the charge? What is its acceleration $\boldsymbol{a}$? What is its total mechanical energy?

(b) What is the rate at which the charge loses energy by radiation?

(c) How long will it take for the orbital radius of the radiating charge to decrease R/e (e is Euler's number)

(d) If the charge had some initial out-of-plane velocity component, explain how your answers to parts (a)-(c) will change, calculating explicitly those changes.

2-Two circularly polarized plane waves of opposite helicity but with the same frequency, wave vector, and amplitude are coherently superposed. Let the phase constants of the two waves be φ_+ and φ_- .

(a) Show by using the complex notation that the resulting plane wave is linearly polarized.

(b) How does the direction of the resulting linear polarization depend on the relative phase, $\Delta\phi = \varphi_+ - \varphi_-$.

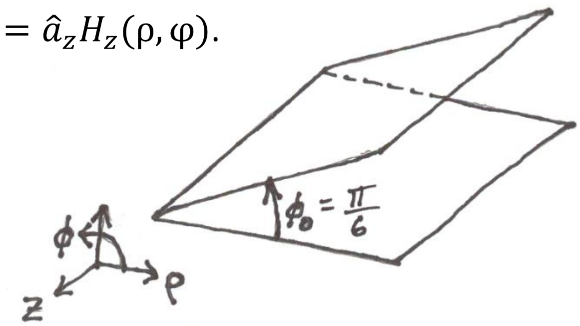
(c) The two waves are allowed to enter an optically active medium, which changes their propagation constants to $\boldsymbol{k}_\pm$, where the two signs refer to the opposite helicities of the wave polarizations. What is the detailed nature of the polarization of the combined wave after a distance L of the propagation in the medium? After what propagation length will the original linear polarization be restored? Express your answers in quantitative terms involving $\boldsymbol{k}_\pm$ and L.

3-A monochromatic plane wave with a angular frequency ω is incident from vacuum at the plane surface of a medium of real electrical conductivity σ and real permittivity ε.

(a) Show that the interaction of the monochromatic wave with the medium may be described accurately in terms of an effective (but complex) index of refraction. Express this index in terms of the given quantities and natural constants, and briefly describe the microscopic basis of the different parts of the expression. What is the physical significance of the complex nature of the index?

(b) What is the characteristic depth within the medium to which the incident wave will penetrate if it is incident at an angle θ to the surface normal. Do not make any assumptions about the ratio, $\varepsilon\omega/\sigma$, in order to simplify your expressions.

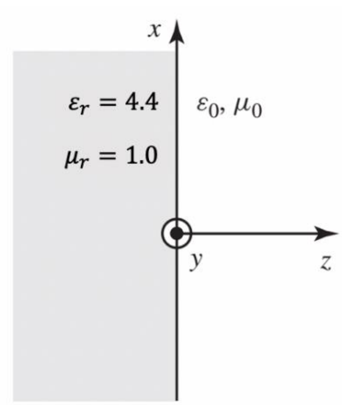
4-A perfectly conducting plate is positioned at an incline above another perfectly conducting plate to form a wedge shaped structure at an angle $\phi_0 = \pi/6$, as shown below. (The bottom plate is at angle $\phi = 0$, and the top plate is at angle $\phi = \phi_0 = \pi/6$). Both plates are infinite in size in the ρ and z directions, and there is free space between them. We are looking for a wave solution in the region inside the wedge. Assume that the magnetic field, *H*, is purely in the z-direction, and is uniform in z. That is, $\vec{H} = \hat{a}_z H_z(\rho, \varphi)$.



a) Write the equation that is satisfied by the scaler H_z.
b) Find the lowest order solution for $\vec{H}$. There may be unknown constants in your final answer.

5-A uniform plane wave is traveling in a dielectric medium with $\varepsilon_r = 4.4$ and $\mu_r = 1.0$ and is incident normally upon free-space via a planar boundary, as shown in the figure below. If the incident electric field is given by $\boldsymbol{E}^i = 2 \times 10^{-3} \hat{\boldsymbol{a}}_y e^{-j\beta z}$ V/m, find:

(a) The corresponding incident magnetic field.
(b) The reflection and transmission coefficients.
(c) The reflected and transmitted electric and magnetic fields.
(d) The incident, reflected, and transmitted power densities.
(e) What is the polarization state of the incident plane wave (specify the direction of the *E*-field)?
(f) Does the reflected electric field flip sign or does it not flip sign?



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1-Describe $\lambda/2$ and $\lambda/4$ waveplates. What is the meaning of the 'order' of the waveplate? What is the importance of the order when using short pulse lasers?
(10 points)

2-A red HeNe laser emitting 1 mW in a beam of 1 mm diameter is aimed at the moon that is 300,000 km away from earth. (10 points)

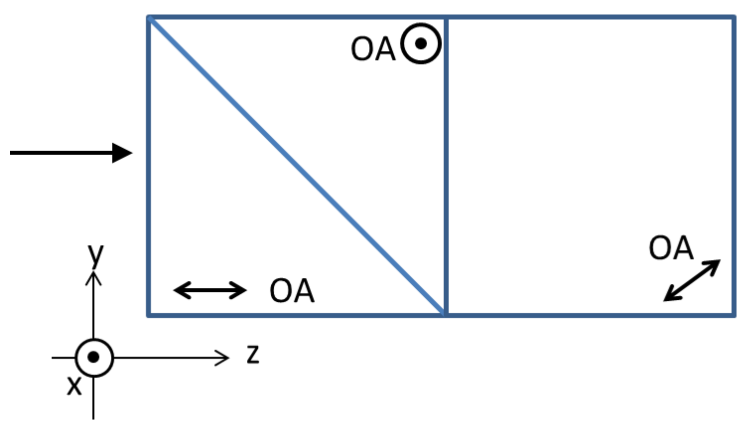
a) What is the approximate diameter of the illuminated region on the moon's surface? (don't guess , estimate)
b) Estimate the number of the photons per second per m^2 that strike the surface of the moon. What is the intensity of the illumination at the moon surface? (neglect losses)

3-Explain in few sentences the operational principle of: (35 points)

a) Acousto-optical modulator.
b) Selecting a single pulse from a train of pulses using a Pockels cell.
c) Generating polarized light using a glass slide.
d) A blazed grating.

4-A non-polarized beam enters a Rochon prism and then another crystal (all made of the same negative uniaxial crystal) as shown below. Evaluate the direction of *k*-vector and Poynting vector for the beam passing through sections with different orientations of optical axis (there are three sections): (10 points)

a) Input beam is polarized along x-direction.
b) Input beam is polarized along y-direction.



5-An object with a height of 3 cm is placed 20 cm from (a) a convex mirror and (b) a concave mirror each with a focal length of 10 cm. Determine the position and nature (inverted/upright) of the image in each case. (10 points)

6- A laser at wavelength of 500 nm provides a TEM_{00} Gaussian beam with a $1/e^2$ waist radius of w_0=0.2 mm. The waist is located at the output coupler. (15 points)

a) The output beam power is 1 mW. What is the maximum intensity, just outside the output coupler?

b) Suppose a lens has a focal length of 8000 mm. In what position(s) could you place this lens so that the beam after passing through it, has a plane wavefront?

c) The lens has now a focal length of 80 mm. Could you place it anywhere, so that the beam after passing through it has a plane wavefront (i.e. is collimated)?

7- The output of a Michelson interferometer is fed to a photodetector: (15 points)

a) The input is a (monochromatic) HeNe laser beam of wavelength λ. If one mirror moves at a speed of *v*, what is the modulation frequency *f* of the photocurrent during the motion?

b) The maximum mirror translation is 10 cm. What is the minimum resolvable optical frequency Δν at the 500 nm if used as a wavemeter? (note that Δν is an estimate of the smallest frequency separation of two monochromatic, mutually incoherent sources near 500 nm that the spectrometer us able to resolve.)

8- A grating consists of alternating opaque and transparent bars, each of width 1 micron. It is illuminated by an expanded, essentially parallel laser beam, λ=500 nm, at normal incidence. You wish to use a lens of focal length f =20 mm to make a 10× magnified image of the grating on a screen: (10 points)

a) Where should the lens and the screen be placed?

b) What is the minimum size (diameter) of the lens that will give an image where the grating bars are clearly distiguishable?

9-Consider an atomic beam of Lithium, irradiated transversely by radiation at 670 nm. Calculate the recoil velocity of one atom that has absorbed a photon of light. Calculate the corresponding change in kinetic energy of that atom. The atomic mass of Lithium is 1.15×10^{-23}g. The recoiling atom sees the 670 nm radiation Doppler shifted by $\Delta\omega_D$. Calculate the Doppler shift and compare $\hbar\Delta\omega_D$ with the kinetic energy of atom. (15 points)

10-A proposed approach for satellite propulsion is to "beam up" energy with a powerful laser, which is then collected by solar (photovolatic) panels, and converted into energy to power an ionic engine to propel the satellite. Another approach is to attach large reflectors to the satellite and to "push" it with the radiaiton pressure from a powerful light beam. (20 points)

a) To get a comparison between the two approaches, consider the first approach with energy conversion efficiency of 10% (from laser light to kinetic energy). Assume a continuous laser beam of 1 MW power, being completely collected by solar panels. Assume a mass of 200 kg, what will be the velocity increase of the spacecraft after 100 s of laser irradiation.

b) Replace the solar panels by perfectly reflecting mirrors. The laser beam is now used to push the spacecraft by radiation pressure. What will be the velocity after 100 seconds (still a 1 MW laser beam)?

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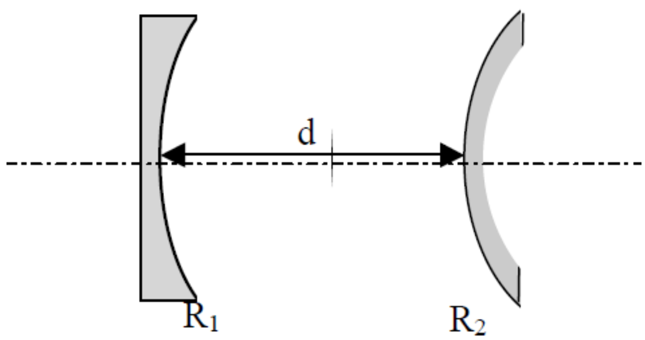
1-Consider the cavity shown below consisting of two spherical mirrors separated by distance *d*. Assume $|R_1|=|R_2|=50$ cm, and $\lambda=1$ µm. (30 points)



a) Find the range ($d_{min} < d < d_{max}$) for which the cavity is stable. (you may use the ABCD matrix or simple logic based on law of reflection)

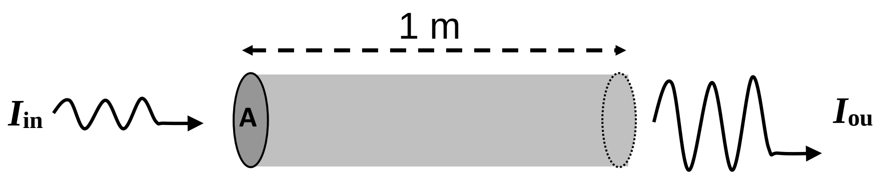
b) Choose d=30 cm and find the location (with respect to one of the mirrors) and magnitude of the beam waist (which may or may not be inside the cavity) for $\lambda=1$ µm. Qualitatively sketch the beam on the above diagram.

c) An external laser beam is to be coupled (matched) into this cavity. This is done by matching the q-parameter (i.e. spot size and curvature) of the incident beam with that of the cavity. If the incident beam is coupled from the left (i.e. mirror 1), what should be the incident beam spot size (w) and curvature (R) at the flat surface of mirror 1(assume thin mirror substrates with n=1.5).

2- A homogeneously broadened optical amplifier with cross-sectional area of A= 0.5 cm^2 and saturation intensity of 20 W/cm^2 has small signal gain of 10 dB.

(30 points)

(note: dB=$\log_{10} I_{out}/I_{in}$)



(a) Calculate the small signal gain coefficient (γ_0).

(b) Estimate the maximum power that can be extracted from this amplifier

(c) If the input power is such that the overall gain (G) is suppressed by a factor of 2 (from its small signal value), compute the output power.

3- The cavity shown below has quality factor (Q) of 3×10^7. The laser utilizes an atomic transition in the active medium that peaks at 0.55 mm with $A_{21}=4\times10^5$ sec-1. The degeneracy of the upper and lower levels is 5 and 3 respectively. The active material has a refractive index of 1.3 with an inhomogeneous line shape that can be approximated by the graph shown below. (40 points)

(a) Estimate the photon lifetime in the passive cavity.
(b) Calculate the stimulated emission cross section for this laser.
(c) Assuming the population of the state-1 is 10^{12} cm^{-3}. Calculate the population of state-2 to reach threshold.
(d) If we pump this laser such that the small signal gain coefficient of the active medium is three times larger than its threshold value, how large is the intensity of the circulating optical power inside the cavity compared to the saturation intensity?